Asymmetric Information and Capital Accumulation*

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Abstract

We study the macroeconomic implications of asymmetric information in capital markets. We build a general-equilibrium capital-accumulation model in which capital is traded in decentralized markets, with sellers having more information about capital quality than buyers. Asymmetric information distorts the terms of trade for sellers of high-quality capital, who list higher prices and are willing to accept lower trading probabilities to signal their type. Led by the model's predictions, we propose an identification strategy to measure the degree of asymmetric information based on the relationship between listed prices and selling probabilities and apply it using a unique dataset on a panel of individual capital units listed for trade. By combining the model and empirical measurement, we show that the degree of asymmetric information has quantitatively large effects on aggregate income levels and operates through three channels: aggregate investment, capital unemployment rate, and average quality of employed capital.

Keywords: Asymmetric information, investment, misallocation, trading frictions.

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1 Introduction

Economic theories have long argued that asymmetric information can play a central role in asset markets, because it affects the quality, valuation, and liquidity of assets traded (e.g., Akerlof, 1970; Stiglitz and Weiss, 1981; Guerrieri, Shimer, and Wright, 2010). In this paper, we study the macroeconomic implications of asymmetric information in physical capital markets. To what extent can the degree of information available to market participants affect an economy's capital accumulation and income levels?

To answer this question, we develop a general-equilibrium capital-accumulation model with asymmetric information in capital markets. Capital is traded in decentralized markets, with capital sellers having more information about capital quality than buyers. Asymmetric information distorts the terms of trade for sellers of high-quality capital, who list higher prices and are willing to accept lower trading probabilities to signal their type. Led by the model's predictions, we propose an identification strategy to measure the degree of asymmetric information based on the relationship between listed prices and selling probabilities. We apply this strategy using a unique dataset on a panel of individual capital units listed for trade, in which we observe the listed price and duration of individual units. By combining the model and empirical measurement, we show that the degree of asymmetric information in capital markets has quantitatively large effects on aggregate income levels and operates through three channels: the aggregate investment, capital unemployment rate, and average quality of employed capital.

Our theoretical framework starts from the environment of a neoclassical growth model, in which households produce capital and firms employ it in production, with three additional key ingredients. First, capital units are heterogeneous in their quality—i.e., in terms of the flow of the final goods units they generate in production. Second, information about capital quality is private to the owner of a capital unit. Third, trading of capital occurs in non-Walrasian markets, with sellers announcing a capital quality and choosing at what price to list their capital units, and buyers choosing at what price and announced quality to search. Buyers have access to an inspection technology that with some probability reveals the true quality of the capital unit. The accuracy of this inspection technology characterizes the degree of asymmetric information in the economy, nesting an economy with full information in the case in which the inspection technology always reveals capital quality. We provide conditions whereby a balanced-growth path is characterized by a unique fully revealing separating equilibrium, in which sellers announce the true quality of their capital. This separating equilibrium resembles that of the classical model of Spence (1973), in which low types have a high marginal cost of effort and choose not to mimic the education levels of high types. In asset markets, the equivalent marginal effort exerted by high types corresponds to selling with a lower probability: Insofar as there is a probability of buyers detecting the true capital quality, high-quality sellers have a lower marginal cost of not trading than low-quality sellers.

To identify the degree of asymmetric information, we show that in our framework the relationship between posted prices and the probability of selling critically depends on the accuracy of buyers' inspection technology, which indexes the degree of information asymmetry. When asymmetric information is low, high-quality capital attracts more buyers and has a higher selling probability than low-quality capital. When the degree of asymmetric information is large, high-quality capital sellers choose to signal their type and separate from sellers of low-quality assets. They do so by choosing to list high-quality capital at such high prices that low-quality assets would not choose to mimic their pricing behavior; higher prices attract fewer buyers and are associated with lower trading probabilities.

Based on the model predictions, we measure the degree of asymmetric information on capital markets using microlevel data on listed prices and time-to-sell of individual capital units. For this, we use a dataset that allows us to construct a novel joint measurement of individual market prices and duration of capital units listed for trade. In particular, our dataset contains the history of nonresidential structures (retail and office space) listed for rent and sale in Spain by one of Europe's main online real estate platforms, Idealista; it contains rich information on each unit, including the listed price, exact location, size, age, and other characteristics. Given the data's panel structure, for each unit we can compute the duration on the platform and the search intensity it attracted, measured by the number of clicks received in a given month.

Our empirical analysis begins by isolating the component of a capital unit's price that reflects the characteristics that are public information by estimating a hedonic regression of (log) prices per square foot on the set of characteristics included in each listing within a narrowly defined market (e.g., a neighborhood) and compute the predicted and residual prices from the hedonic regression. We then study the comovement between predicted and residual prices with duration on the market. The data show (i) a negative relationship between predicted prices and duration and (ii) a positive relationship between residual prices and duration. The first empirical fact validates the prediction of the model under full information: Since predicted prices are obtained from observable characteristics, the theory predicts that on average, properties with better characteristics (which are reflected by a higher predicted price) should have a shorter duration on the market. The second fact is consistent with the theory's prediction regarding capital quality under asymmetric information.

We also provide a set of empirical results showing that our findings would be hard to rationalize using other theories of trading in asset markets that do not explicitly incorporate information frictions. First, we examine whether theories of price dispersion in markets with search frictions (e.g., Burdett and Judd, 1983) could rationalize our empirical findings. In principle, these theories generate a positive relationship between residual prices and duration: Sellers of homogeneous properties are indifferent between selling quickly at a low price or waiting in order to sell at a higher price, and thus randomize their choices. However, given the quantitative relation between residual prices and duration in the data, we show that any seller facing such a price-duration trade-off will maximize expected discounted revenues by choosing the highest price we see in the data. Second, we analyze whether heterogeneous sellers' preferences could rationalize the empirical fact. To do this, we repeat the analysis by computing the expected discounted revenues for a broad set of preferences (discount factors from 0 to 0.99 and attitude toward risk from risk neutral to extreme forms of risk aversion). We find that all types of sellers would maximize their expected net present value by choosing the highest price observed in the data. Finally, we examine whether the heterogeneous holding costs that sellers must pay could explain this fact, and conclude that in order for differential holding costs to explain the differences in expected discounted revenues in the data, they must be extremely large. It is also worth noting that none of these alternative theories can rationalize the negative relationship between predicted prices and duration simultaneously with a positive relationship between residual prices and duration.

Finally, we combine the model and empirical measurement to quantify the extent of asymmetric information in capital markets and study its aggregate implications. We do so by calibrating the parameters of the model linked to information frictions—i.e., the dispersion of unobserved capital qualities and the accuracy of buyers' inspection technology—to match the standard deviation of residual prices and their covariance with duration. Then, with our parameterized model, we quantify the effects of asymmetric information by comparing the estimated model's predictions with the prediction of a model in which there is no private information. Our exercise indicates that asymmetric information has large effects on economic activity. This occurs through three channels. First, lower information asymmetries lead to a higher capital stock. This is because lower information asymmetries are associated with a higher revenue for sellers of high-quality capital, which increases the returns to producing capital goods. Second, lower information asymmetries lead to a lower unemployment rate of capital. As information asymmetries decrease, so do the listed prices of high-quality capital sellers, which increases the selling probability and reduces the duration in unemployment of listed units. Third, a lower degree of asymmetric information is associated with a higher average quality of employed capital. This is because information asymmetries disproportionally affect the allocation for sellers of high-quality capital, who have to prevent mimicking by lower types through higher prices and lower trading probabilities. Overall, a lower degree of asymmetric information in the economy is associated with a higher steady-state income per capita. For example, long-run income per capita in an economy with no private information would be more than 10% larger than in an economy with the estimated degree of asymmetric information.

Related Literature First, our paper is related to the literature that studies asymmetric information in asset markets, pioneered by Akerlof (1970); Stiglitz and Weiss (1981); and Myers and Majluf (1984), among others. Our framework particularly builds on theories that study these frictions in decentralized markets (see, for example, Guerrieri et al., 2010; Delacroix and Shi, 2013). Our paper is also related to the body of work on the effects of asymmetric information in the macroeconomy (see, for example, Eisfeldt, 2004; Kurlat, 2013; Guerrieri and Shimer, 2014; Bigio, 2015; Lester, Shourideh, Venkateswaran, and Zetlin-Jones, 2019). We contribute to this literature by showing how microlevel data on assets listed for trade can be used to measure the degree of asymmetric information and by using this measurement to show that information asymmetries can have a quantitatively important role in long-run capital accumulation and income levels.

Second, our paper is related to the literature that studies the role of search-and-matching frictions in asset markets. This includes a large body of work on financial markets (for a recent survey, see Lagos, Rocheteau, and Wright, 2017); housing markets (see, for example, Wheaton, 1990; Krainer, 2001; Caplin and Leahy, 2011; Piazzesi, Schneider, and Stroebel, 2020); and physical capital markets (see, for example, Kurmann and Petrosky-Nadeau, 2007; Gavazza, 2011; Cao and Shi, 2017; Ottonello, 2017; Wright, Xiao, and Zhu, 2018, 2020; Cui, Wright, and Zhu, 2021). Our paper contributes to this literature by showing the relevance of the interaction between asymmetric information and search frictions.

Third, the paper is related to the literatures on misallocation (e.g., Hsieh and Klenow, 2009; Restuccia and Rogerson, 2008); capital reallocation (e.g., Ramey and Shapiro, 2001; Eisfeldt and Rampini, 2006; Lanteri, 2018); and asset specificity (e.g., Caballero and Hammour, 1998; Kermani and Ma, 2020). We contribute to this literature by showing that asymmetric information can constitute a sizable source of asset specificity and misallocation.[1]

Fourth, our paper is related to the literature that measures asymmetric information in insurance markets (e.g., Chiappori and Salanie, 2000); financial markets (see, for example, Ivashina, 2009;

[1]The empirical literature on misallocation has traditionally focused on studying how capital is distributed across active establishments and firms (see, for example, Alfaro, Charlton, and Kanczuk, 2008; Bartelsman, Haltiwanger, and Scarpetta, 2013; Midrigan and Xu, 2014; Gopinath, Kalemli-Özcan, Karabarbounis, and Villegas-Sanchez, 2017; Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry, 2018; David and Venkateswaran, 2019; Kehrig and Vincent, 2019; Hopenhayn, 2014, and references therein). Our study highlights a complementary source of capital misallocation that arises between active firms and other agents in the economy that hold capital but are not actively producing (e.g., unemployed capital from exiting firms, which is usually not included in establishment-based measures of misallocation). The form of misallocation we consider builds on that in Gavazza (2016), who uses business-aircraft data to study the welfare effects of trading frictions in the allocation of assets.

Einav, Finkelstein, and Schrimpf, 2010); and housing markets (see, for example, Kurlat and Stroebel, 2015). Our paper complements these studies by developing a methodology to measure asymmetric information that exploits the relationship between prices and trading probabilities—which typically characterize asset markets—that can be applied more broadly to other frictional markets.

Layout The rest of the paper is organized as follows. Section 2 presents the theoretical framework. Section 3 presents the data and empirical measurement of information frictions. Section 4 maps the model to the data and quantifies the aggregate effects of informational asymmetries. Section 5 concludes.

2 Theoretical Framework

We construct a model that embeds information and trading frictions into the standard neoclassical capital accumulation framework. We use the model to show how the degree of information frictions affects macroeconomic aggregates and how it can be identified from the micro-data.

2.1 Environment

Time is discrete and infinite, and there is no aggregate uncertainty. Final goods are perishable and can be used for consumption or investment. Capital goods are storable and can be used in the production of final goods, together with labor services.

Agents, preferences, and technology The economy is populated by a unit mass of identical households and a unit mass of firms owned by the household. Households have preferences over consumption described by the lifetime utility function $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t)\gamma_n^t$, where c_t denotes per capita consumption in period t; $\gamma_n \geq 1$ denotes the gross population within the representative household;[2] $u : \mathbb{R}_+^2 \to \mathbb{R}$ is a continuous, differentiable, increasing, and concave function; $\beta \in (0, 1)$ is the subjective discount factor; and $\mathbb{E}_t$ denotes the expectation conditional on the information set available in period t. Households are endowed with $\bar{h}$ hours of work each period, which they supply inelastically. In addition, households have access to a linear technology to produce new capital goods using final goods.

A continuum of identical nonfinancial firms with measure one have access to a constant-returns-to-scale technology to produce final goods using capital and labor as inputs, $y_{jt} = f_t(\mathcal{K}_{jt}, l_{jt}) \equiv \mathcal{K}_{jt}^{\alpha}(\gamma^t l_{jt})^{1-\alpha}$, where y_{jt}, $\mathcal{K}_{jt}$, and l_{jt} denote the output, capital input, and labor input of firm j in

[2] We include population and technology growth in the model to better match the investment rates observed in the data, which are sizable flows for capital markets.

period t, respectively; $\gamma \geq 1$ denotes the exogenous growth rate of labor-augmenting technology in the economy, and $\alpha \in (0,1)$. Each period, with i.i.d. probability φ, each firm receives an exit shock and must exit the economy; firms that exit the economy cannot produce, and transfer their capital to households at the end of the period. After exit shocks are realized, a new mass φ of firms enter the economy. In this setup, operating firms will be capital buyers and households capital sellers (selling new capital or capital from exiting firms).

The model features three main departures relative to the standard neoclassical capital-accumulation model: heterogeneity in capital quality, a decentralized market for capital, and information frictions. We describe each of these elements next.

Capital-quality heterogeneity Studying information asymmetries in capital markets requires introducing heterogeneity in these goods. To do so, we consider an environment in which the capital stock is composed of infinitesimal indivisible units (i.e., capital goods are available to trade in integer quantities only, and agents hold a mass of these units). Capital units are heterogeneous in two dimensions: an "observed quality" $\omega \in \Omega \equiv [\omega_1, \ldots, \omega_{N_\omega}]$, with $\omega_r < \omega_s$ for $r < s$, and an "unobserved quality" $a \in \mathcal{A} \equiv [a_1, \ldots, a_{N_a}]$, with $a_r < a_s$ for $r < s$. While the observed quality ω of a unit is assumed to be perfectly observable by all market participants, unobserved quality a is the private information of the owner of the capital unit and is the source of asymmetric information in the model, which is further discussed below. The capital services a capital unit provides are determined by these qualities, with the capital services of a capital unit i being given by $\omega_i a_i$. Capital services employed as input in production by firm j are then given by $\mathcal{K}_{jt} = \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} \omega a k_{jt+1}(\omega, a)$, where $k_{jt+1}(\omega, a)$ is the mass of capital of quality (ω, a) employed in production by firm j in period t. In our application, we interpret capital quality broadly, representing any characteristic that increases the marginal product of capital. For example, in the case of retail space, capital quality can capture the number of potential clients in a given geographic location or the quality of the store in attracting customers.

Decentralized capital market In models of asymmetric information, the presence of search-and-matching frictions can play an important role in determining the equilibrium, because it introduces a signaling device for sellers in choosing a price that reflects their different marginal benefits of trading (see, for example, Guerrieri et al., 2010). Based on the empirical evidence of studies that characterize trading capital markets (e.g., Gavazza, 2011; Ottonello, 2017), we assume that capital goods are traded in a decentralized market with search-and-matching frictions.

The decentralized capital market is organized in a continuum of submarkets, indexed by $(\omega, \hat{a}, q)$, where ω is the observed quality, $\hat{a}$ is the unobserved quality announced by the seller, and q is the listed price. Search is directed: Sellers can choose at what announced unobserved quality and price to list their capital units, and buyers can choose at what observed quality, announced unobserved quality, and price to search, dedicating labor to search and match.[3] When sellers list a capital unit in submarket $(\omega, \hat{a}, q)$, they commit to allow potential buyers to inspect the unit using a technology further described below. If no new information about the capital quality is revealed during the inspection or if the inspection indicates that capital quality is not below that announced (i.e., $a' \geq \hat{a}$), sellers and buyers commit to trade the capital unit at the listed price of q. If the inspection reveals that the quality of the capital is some $a' < \hat{a}$ and there are gains from trade between the buyer and seller (formally defined below), trade occurs at the inspection-adjusted price $q_t^P(\omega, a', \hat{a}, q) \leq q$. The inspection-adjusted price function $q_t^P : \Omega \times \mathcal{A}^2 \times \mathbb{R}_+ \to \mathbb{R}_+$ is assumed to be nondecreasing in the revealed quality, i.e., $\forall (a, a') \in \mathcal{A}^2$ such that $a' > a$, $\forall (\omega, \hat{a}, q) \in \Omega \times \mathcal{A} \times \mathbb{R}_+ : q_t^P(\omega, a', \hat{a}, q) \geq q_t^P(\omega, a, \hat{a}, q)$, and nests the case of bargaining between the buyer and seller, which we consider in our quantitative analysis; if there are no gains from trade for quality $a' < \hat{a}$ the match is dissolved without trade.

In each submarket $(\omega, \hat{a}, q)$, the market tightness, denoted by $\theta_t(\omega, \hat{a}, q)$, is defined as the ratio between buyers' hours of search and the mass of capital posted by sellers.[4] Visiting submarket $(\omega, \hat{a}, q)$ in period t, sellers face a probability $p(\theta_t(\omega, \hat{a}, q))$ of finding a potential buyer for their unit and buyers match with a mass $\mu_t(\theta_t(\omega, \hat{a}, q))$ of potential units to buy per hour of search, where $p(\theta) = \min\{\overline{m}\theta^{1-\eta}, 1\}$ with $\eta \in (0, 1)$.[5] Finally, since our main focus is on capital markets, we assume that final goods and labor services are traded in Walrasian markets.

Information structure An information asymmetry arises because capital quality has a component that is private information to its owner, a_i. We are interested in studying how the *degree* of asymmetric information in the economy affects capital accumulation. For this, we assume that after having searched and matched with a capital unit and before purchasing it, buyers have access to a technology to inspect the unit. Similar to Menzio and Shi (2011a), this inspection technology is such that in any submarket $(\omega, \hat{a}, q)$, there is a probability ψ that the buyer learns the true type (ω, a) of the capital good and a probability $1 - \psi$ that the inspection is uninformative. Hence, ψ parameterizes the degree of asymmetry of information in the economy, nesting the full-information case when

[3] The assumed directed search structure is similar to that used by Shimer (1996); Moen (1997a); and Menzio and Shi (2011a) in the labor market, and Ottonello (2017) in capital markets. For a recent survey of the literature on directed search in labor, housing, and monetary economics, see Wright, Kircher, Julien, and Guerrieri (2019).

[4] Following the directed search literature (see, for example Moen, 1997b; Menzio and Shi, 2011b), in submarkets that are not visited by any sellers, $\theta_t(\omega, \hat{a}, q))$ is an out-of-equilibrium conjecture that helps determine equilibrium.

[5] The functional form of the matching probability can be obtained from a Cobb-Douglas matching technology $\mathcal{M}_t(k^s(\omega, \hat{a}, q), \gamma^t v^s(\omega, \hat{a}, q)) = \min\{\overline{m}(k^s(\omega, \hat{a}, q))^\eta(\gamma^t v^s(\omega, \hat{a}, q))^{1-\eta}, k^s(\omega, \hat{a}, q)\}$, where $k^s(\omega, \hat{a}, q)$ and $v^s(\omega, \hat{a}, q)$ denote the mass of capital listed by sellers and hours dedicated by buyers to search in submarket $(\omega, \hat{a}, q)$, respectively, and $\overline{m} > 0$; given the labor-augmenting technology in the production of final goods, the labor-augmenting technology in the matching sector is necessary for a balanced-growth path.

$\psi = 1$, and the case with complete asymmetric information when $\psi = 0$ (since there cannot be any discovery of the unobserved quality).

The information asymmetry requires that we specify agents' beliefs about the type of capital available for sale, given a listed price and observable characteristics. We assume that all potential buyers have the same beliefs. We describe beliefs by the mapping $\pi_t(a|\omega, \hat{a}, q) : \Omega \times \mathcal{A}^2 \times \mathbb{R}_+ \to [0,1]$, which denotes the probability that a unit of capital is of unobserved type a, given the observed type ω, the announced quality $\hat{a}$, and the price q. After purchasing a unit of capital, buyers receive its ownership and obtain full information about its quality. Sellers are assumed not to have recall on the capital quality of their units sold.

Timing The timing of events within each period is as follows:

i. Exit shocks are realized, and a mass φ of new firms enter the economy.

ii. Households choose the capital units they list for sale, their prices, and their announced qualities, which are perfectly observed by all agents. Incumbent non-exiting firms and new firms search and match with potential capital units to buy.

iii. Firms conduct inspections of matched capital units and decide whether to buy them or not.

iv. Incumbent non-exiting firms and new firms hire workers, produce final goods, and pay wages. Firms that exit the economy transfer their capital to households. All agents holding capital pay a maintenance cost δ per unit of effective capital in terms of final goods. Households invest in new capital units and consume.

2.2 Optimization

Households Each period, households produce new capital goods. They do so by choosing their total investment in terms of final goods i_t, and the resulting quality of new capital is exogenous and random, governed by the distribution function $g : \Omega \times \mathcal{A} \to [0,1]$, which describes the measure of new capital of each quality. Since households do not have access to a production technology, their capital revenue comes from selling these newly produced units of capital, together with unemployed capital transferred by exiting firms, to operating firms. The evolution of the capital holdings by households are then given by

$$k_{Ht+1}(\omega, a) = (1 - p(\theta_t(\omega, \hat{a}_{Ht}(\omega, a), q_{Ht}(\omega, a)))k_{Ht}(\omega, a) + g(\omega, a)i_t + \varphi K_{Ft}(\omega, a), \tag{1}$$

where $k_{Ht+1}(\omega, a)$ denotes capital of quality (ω, a) held by the household at the end of period t, $\hat{a}_{Ht}(\omega, a)$ and $q_{Ht}(\omega, a)$ denote the household's choice of announced capital quality and price to list units of quality (ω, a); $p(\theta_t(\omega, \hat{a}_{Ht}(\omega, a), q_{Ht}(\omega, a))k_{Ht}(\omega, a)$ denotes the mass of capital of type (ω, a) matched by buyers given household's choice of submarket; and $K_{Ft}(\omega, a)$ denotes the aggregate capital of quality (ω, a) held by firms at the beginning of period t—a fraction φ of which is transferred to households by firms that exit. For expositional simplicity, equation (1) abstracts from households posting a unit of capital in multiple submarkets and from capital not being sold following an inspection that reveals a different quality from that announced (Appendix A shows that the latter does not happen in equilibrium).

We write households' optimization problem recursively. At the beginning of a period, the individual state for the household is a matrix of its capital holdings, given by $\mathbf{k} \equiv \begin{bmatrix} k(\omega_1, a_1) & ... & k(\omega_{N_\omega}, a_1) \\ ... & ... & ... \\ k(\omega_1, a_{N_a}) & ... & k(\omega_{N_\omega}, a_{N_a}) \end{bmatrix}$. The recursive problem of the representative household is then given by

$$V_{Ht}(\mathbf{k}) = \max_{\{c, \{k'(\omega,a), \hat{a}(\omega,a), q(\omega,a)\}, i \geq 0\}} u(c)\gamma_n^t + \beta V_{Ht+1}(\mathbf{k}'), \tag{2}$$

subject to:

$$c\gamma_n^t + i + \delta \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} \omega a \left((1 - p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a))k(\omega, a) + \varphi K_{Ft}(\omega, a)\right)$$

$$= \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} \left[(1-\psi)q(\omega, a) + \psi q^P(\omega, a, \hat{a}(\omega, a), q)\right] p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a))k(\omega, a) + w_t \bar{h}\gamma_n^t + Div_{Ft}$$

$$k'(\omega, a) = (1 - p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a))k(\omega, a) + ig(\omega, a) + \varphi K_{Ft}(\omega, a),$$

where Div_{Ft} denote the dividends transferred by firms in period t. The optimal level of investment (provided that $i > 0$) is characterized by the Euler equation

$$1 = \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} g(\omega, a)\lambda_t(\mathbf{k})\nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{k}),$$

where $\lambda_t(\mathbf{k}) \equiv \beta\gamma_n \frac{u_{ct+1}(\mathbf{k}_{Ht+1}(\mathbf{k}))}{u_{ct}(\mathbf{k})}$ ($\mathbf{k}_{Ht+1}(\mathbf{k})$ is the matrix of policy functions for capital accumulation associated with problem (2)) and $\nu_t^{\mathrm{s}}(\omega, a, \mathbf{k}) \equiv \frac{\partial V_{Ht}(\mathbf{k})}{\partial k(\omega,a)} \frac{1}{u_{ct}(\mathbf{k})\gamma_n^t}$ is the household's marginal value of capital of type (ω, a) measured in final goods, which satisfies the recursive problem:

$$\nu_t^{\mathrm{s}}(\omega, a, \mathbf{k}) = \max_{\hat{a}, q} \; p(\theta_t(\omega, \hat{a}, q)((1-\psi)q + \psi q^P(\omega, a, \hat{a}, q)) \\ + (1 - p(\theta_t(\omega, \hat{a}, q)))\left(\lambda_t(\mathbf{k})\nu_{t+1}^{\mathrm{s}}(\omega, a, k_{Ht+1}(\mathbf{k})) - \delta\omega a\right). \tag{3}$$

Firms Firms accumulate capital by purchasing it from sellers in the decentralized market, which requires paying for hours of labor to search for potential units that are a good match for the firm. Abstracting from the possibility that firms might want to sell capital (which does not occur in equilibrium), the evolution of their capital holdings is given by

$$k_{jt+1}(\omega, a) = \sum_{\hat{a} \in \mathcal{A}} \int_{q \in \mathbb{R}_+} \iota_t(a|\omega, \hat{a}, q) \mu_t\left(\theta\left(\omega, \hat{a}, q\right)\right) v_{jt}(\omega, \hat{a}, q)\, \mathrm{d}q + k_{jt}(\omega, a), \tag{4}$$

where $v_{jt}(\omega, \hat{a}, q)$ denotes the hours of work hired by firms to search and match with sellers in submarket $(\omega, \hat{a}, q)$, $\mu_t\left(\theta_t\left(\omega, \hat{a}, q\right)\right) v_{jt}(\omega, \hat{a}, q)$ the mass of capital matched by these workers, and $\iota_t(a|\omega, \hat{a}, q)$ the share of units of capital of quality a when searching in submarket $(\omega, \hat{a}, q)$ in period t.

Conditional on not exiting, the value of the firm solves the Bellman equation

$$V_{Ft}(\mathbf{k}) = \max_{\{l, \{v(\omega,\hat{a},q) \geq 0\}, \{k'(\omega,a)\}\}} \mathbb{E}_a[div + \Lambda_{t,t+1}((1-\varphi)V_{Ft+1}(\mathbf{k}') + \varphi V^{\text{exit}}_{t+1}(\mathbf{k}'))], \tag{5}$$

such that

$$div = \left(\sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} \omega a k'(\omega, a)\right)^{\alpha} (\gamma^t l)^{1-\alpha} - w_t l - \delta \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} \omega a k'(\omega, a)$$

$$- \sum_{\omega \in \Omega} \sum_{\hat{a} \in \mathcal{A}} \int_{q \in \mathbb{R}_+} [(\psi \sum_{a \in \mathcal{A}} \iota_t(a|\omega, \hat{a}, q) q^P_t(\omega, a, \hat{a}, q) + (1-\psi) q) \mu_t\left(\theta\left(\omega, \hat{a}, q\right)\right) + w_t] v(\omega, \hat{a}, q)\, \mathrm{d}q$$

$$k'(\omega, a) = \sum_{\hat{a} \in \mathcal{A}} \int_{q \in \mathbb{R}_+} \iota_t(a|\omega, \hat{a}, q) \mu_t\left(\theta\left(\omega, \hat{a}, q\right)\right) v(\omega, \hat{a}, q)\, \mathrm{d}q + k(\omega, a),$$

where $\mathbb{E}_a$ denotes the expectation under the belief function $\pi_t(a|\omega, \hat{a}, q)$, div denotes dividends in terms of final goods transferred to households, $\Lambda_{t,t+1}$ denotes households' discount factor (further described below), w_t denotes the wage rate in period t, and $V^{\text{exit}}_t(\mathbf{k}) \equiv \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} k(\omega, a) \nu^{\text{s}}_t(\omega, a, \mathbf{K}_{Ht})$ denotes the household's value of exiting firms with capital matrix $\mathbf{k}$ and $\mathbf{K}_{Ht}$ denotes the matrix of capital stocks by households in period t, which is taken as given by individual firms. Problem (5) abstracts from the scenario in which trade does not occur when there is an inspection and there are no gains from trade for quality $a' < \hat{a}$ (Appendix A provides conditions for $q^P_t(\omega, a, \hat{a}, q)$ for which this does not happen in equilibrium).

The following result characterizes firms' optimal choices of capital and labor.

***Proposition* 1.** *The firm's value function $V_{Ft}(\mathbf{k})$ is linear in capital stocks, i.e., it can be expressed as $V_{Ft}(\mathbf{k}) = \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} \nu^{\text{b}}_t(\omega, a) k_t(\omega, a)$. This marginal value of capital holdings satisfy the recursive* *problem:*

$$\nu_t^{\mathrm{b}}(\omega, a) = (Z_t - \delta)\omega a + \Lambda_{t,t+1}\left[(1-\varphi)\nu_{t+1}^{\mathrm{b}}(\omega, a) + \varphi \nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{K}_{Ht+1})\right], \tag{6}$$

where $Z_t \equiv \alpha\left(\frac{\gamma^t(1-\alpha)}{w_t}\right)^{\frac{1-\alpha}{\alpha}}$. *The labor demand in the production of final goods is linear in* $\mathcal{K}_t$ *and given by* $l_t(\mathcal{K}_t) = \mathcal{K}_t \times \left(\frac{(1-\alpha)\gamma^{t(1-\alpha)}}{w_t}\right)^{\frac{1}{\alpha}}$.

Proof. See Appendix A.1. ■

Proposition 1 implies that the value of a capital unit of a given quality for a buyer does not depend on other capital holdings. In particular, the value of capital with quality (ω, a) is given by the utility flow generated by its production plus its continuation value, which takes into account the probability of exiting production and becoming a seller of capital.

Firms' optimal search activity across different submarkets is characterized by

$$v_t(\omega, \hat{a}, q)\left(\underbrace{((1-\psi)q + \psi \mathbb{E}_a(q^P(\omega, a, \hat{a}, q)|\omega, \hat{a}, q))}_{\text{Expected price}} + \underbrace{\frac{w_t}{\mu_t(\theta(\omega, \hat{a}, q))}}_{\text{Search cost}} - \underbrace{\mathbb{E}_a\left(\nu_t^{\mathrm{b}}(\omega, a)|\omega, \hat{a}, q\right)}_{\text{Expected value}}\right)^+ = 0, \tag{7}$$

for all $(\omega, \hat{a}, q)$, which shows that firms are willing to search for capital in a submarket if the expected marginal cost of purchasing capital in that market, including its expected price and search cost, does not exceed its expected value. Given that submarkets differ in their price q, firms are only indifferent between buying in different submarkets for the same expected value of capital if those units with higher price have associated a higher rate $\mu_t(\omega, \hat{a}, q)$ at which the workers find a match.

2.3 Equilibrium

We define the equilibrium as follows.[6]

Definition* 1. *Competitive Equilibrium

Given initial conditions $\mathbf{K}_{H0}$ *and* $(\mathbf{k}_{j0})_{j\in[0,1]}$*, a perfect Bayesian equilibrium under asymmetric information consists of a sequence of household value functions* $\{V_{Ht}(\mathbf{k}), \nu_t^{\mathrm{s}}(\omega, a, \mathbf{k})\}$ *and policy functions* $\{c_t(\mathbf{k}), i_t(\mathbf{k}),\ \mathbf{k}_{Ht+1}(\mathbf{k}), \hat{a}_t(\omega, a, \mathbf{k}), q_t(\omega, a, \mathbf{k})\}$*; firm value functions* $\{V_{Ft}(\mathbf{k}), \nu_t^{\mathrm{b}}(\omega, a)\}$ *and policy functions* $\{l_t(\mathbf{k}), div_t(\mathbf{k}), \mathbf{k}_{Ft+1}(\mathbf{k}), \{v_t(\omega, \hat{a}, q)\}\}$*; market tightness functions* $\{\theta_t(\omega, \hat{a}, q)\}$*; belief functions* $\{\pi_t(a|\omega, \hat{a}, q)\}$*; wages* $\{w_t\}$*; discount factors* $\{\Lambda_{t,t+1}\}$*; and aggregate variables* $\{\mathbf{K}_{Ht+1}, \mathbf{K}_{Ft+1}, Div_{Ft}, \iota_t(a|\omega, \hat{a}, q)\}$ *for all* $t \geq 0$ *such that*

[6] We restriction attention to pure strategy equilibria, which characterize the unique solution under the D1 equilibrium refinement of Cho and Kreps (1987).

(i) Given wages and market tightness, household's value functions $V_{Ht}(\mathbf{k})$ and $\nu_t^{\mathrm{s}}(\omega, a, \mathbf{k})$ solve (2) and (3) with associated policy functions $c_t(\mathbf{k})$, $i_t(\mathbf{k})$, $\mathbf{k}_{Ht+1}(\mathbf{k})$, $\hat{a}_t(\omega, a, \mathbf{k})$, and $q_t(\omega, a, \mathbf{k})$ for all $(\omega, a) \in \Omega \times \mathcal{A}$.

(ii) Given wages, market tightness, and discount factors, a firm's value functions $V_{Ft}(\mathbf{k})$ and $\nu_t^{\mathrm{b}}(\omega, a)$ solve (5) and (6) with associated policy functions $l_t(\mathbf{k})$, $\mathbf{k}_{Ft+1}(\mathbf{k})$, and $\{v_t(\omega, \hat{a}, q)\}$ for all $(\omega, \hat{a}) \in \Omega \times \mathcal{A}$.

(iii) Market tightness functions satisfy (7) in all submarkets.

(iv) The belief function $\pi_t(a|\omega, \hat{a}, q)$ is consistent with sellers' strategies using Bayes' rule when possible.

(v) The labor market clears: $\sum_{\omega\in\Omega}\sum_{a\in\mathcal{A}}\int_{q\in\mathbb{R}_+} v_t(\omega, \hat{a}, q)\,\mathrm{d}q + \int l_t(\mathbf{k}_{jt})\,\mathrm{d}j = \bar{h}\gamma_n^t$.

(vi) The discount factors satisfy $\Lambda_{t,t+1} = \lambda_t(\mathbf{K}_{Ht})$.

(vii) Aggregate variables are consistent with individual policies: $\mathbf{K}_{Ht+1} = \mathbf{k}_{Ht+1}(\mathbf{K}_{Ht})$, $\mathbf{K}_{Ft+1} = \int \mathbf{k}_{Ft+1}(\mathbf{k}_{jt})\,\mathrm{d}j$, $Div_{Ft} = \int div_t(\mathbf{k}_{jt})\,\mathrm{d}j$; $\iota_t(a|\omega, \hat{a}, q) = \frac{\mathbb{I}_{\{\hat{a}=\hat{a}_t(\omega,a,\mathbf{K}_{Ht})\}}\mathbb{I}_{\{q=q_t(\omega,a,\mathbf{K}_{Ht})\}}K_{Ht}(\omega,a)}{\sum_{a_j\in\mathcal{A}}\mathbb{I}_{\{\hat{a}=\hat{a}_t(\omega,a_j,\mathbf{K}_{Ht})\}}\mathbb{I}_{\{q=q_t(\omega,a_j,\mathbf{K}_{Ht})\}}K_{Ht}(\omega,a_j)}$ for all $(\omega, \hat{a}, q)$ such that $\hat{a}$ and q are part of the set of policy functions associated with the household's problem.

In what follows, we restrict our attention to the balanced-growth path equilibrium, which is defined as:

Definition 2. balanced-growth path

A balanced-growth path is defined as a competitive equilibrium in which the sequence $\{c_t, k_{Ht}(\omega, a), k_{Ft}(\omega, a), \hat{a}_t(\omega, a), q_t(\omega, a), \theta_t(\omega, \hat{a}_t, q_t), w_t, Z_t, \Lambda_{t,t+1}\}_{t\geq 0}$ satisfies:

(i) Per-capita consumption c_t, wages w_t and productivity Z_t grow at rate γ.

(ii) For all (ω, a), the stock of capital held by firms and households ($k_{Ft}(\omega, a)$ and $k_{Ht}(\omega, a)$, respectively) grows at rate $\gamma\gamma_n$.

(iii) For all (ω, a), submarket choices $a_t(\omega, a)$ and $q_t(\omega, a)$ and market tightness $\theta_t(\omega, \hat{a}_t, q_t)$ are constant.

2.4 Characterization of equilibrium

Here, we characterize the solution to the seller's problem, which determines the terms of trade in the capital market. As in Guerrieri et al. (2010), we characterize this solution with the following sequence of constrained optimization problems.

Definition 3. *For a given observed quality ω and aggregate variables, the solution to problem $\mathcal{P}_j(\omega)$ is a vector $(q(\omega,a_j),\hat{a}(\omega,a_j))$ that solves*

$$\nu^{\mathrm{s}}(\omega,a_j)=\max_{\{q(\omega,a_j),\hat{a}(\omega,a_j)\}} p\left(\theta\left(\omega,\hat{a}(\omega,a_j),q(\omega,a_j)\right)\right)\left((1-\psi)q(\omega,a_j)+\psi q^{P}(\omega,a_j,\hat{a}(\omega,a_j),q)\right) \tag{8}$$
$$+\left(1-p(\theta(\omega,\hat{a}(\omega,a_j),q(\omega,a_j)))\right)\left(\beta\gamma_n\nu^{\mathrm{s}}(\omega,a_j)-\delta\omega a_j\right)$$

subject to

$$\theta(\omega,\hat{a}(\omega,a_j),q(\omega,a_j)) = \mu^{-1}\left(\frac{w_t}{\mathbb{E}_a\left(\nu^{\mathrm{b}}(\omega,a)-(1-\psi)q(\omega,a_j)-\psi q^{P}(\omega,a,\hat{a}(\omega,a_j),q)|\omega,\hat{a}(\omega,a_j),q(\omega,a_j)\right)}\right), \tag{9}$$

and

$$\nu^{\mathrm{s}}(\omega,a_{j'})\geq p\left(\theta\left(\omega,\hat{a}(\omega,a_j),q(\omega,a_j)\right)\right)\left((1-\psi)q(\omega,a_j)+\psi q^{P}(\omega,a_{j'},\hat{a}(\omega,a_j),q)\right) \tag{10}$$
$$+\left(1-p(\theta(\omega,\hat{a}(\omega,a_j),q(\omega,a_j)))\right)\left(\beta\nu^{\mathrm{s}}(\omega,a_{j'})-\delta\omega a_{j'}\right)\quad \text{for all } j'<j.$$

In problem $\mathcal{P}_j(\omega)$, the seller of capital with quality (ω,a_j) announces an unobserved quality $\hat{a}(\omega,a_j)$ and posts a price $q(\omega,a_j)$ to maximize expected revenues subject to two constraints. The first constraint is the buyer's search optimality condition, which pins down the market tightness for a given set of beliefs and seller's choices. In addition, the seller is constrained by a set of no-mimicking conditions, which require that sellers of lower quality weakly prefer their chosen terms of trade rather than mimicking the terms of trade chosen by the seller of unobserved quality a_j.

In the setup considered, there could be different types of solutions to the sequence of problems $\{\mathcal{P}_1(\omega),\ldots,\mathcal{P}_{N_a}(\omega)\}$ and corresponding equilibria, each supported by appropriate out-of-equilibrium beliefs. Of particular interest are fully revealing separating equilibria and pooling equilibria, which are defined as follows.

Definition 4. Types of equilibria
A pooling equilibrium is a competitive equilibrium in which sellers of different unobserved qualities post the same price and announce the same quality with strictly positive probability—i.e., $q(\omega,a_j)=q(\omega,a_{j'})$ and $\hat{a}(\omega,a_j)=\hat{a}(\omega,a_{j'})$. Similarly, a fully revealing separating equilibrium is a competitive equilibrium in which sellers of different unobserved qualities post different prices and announce their true qualities—i.e., $q(\omega,a_j)=q(\omega,a_{j'})$ and $\hat{a}(\omega,a_j)=a_j$.

Our focus is on equilibria in which sellers of different unobserved qualities either participate in the same submarket with positive probability or choose to truthfully announce their quality and set different prices.[7] The definition of a fully revealing separating equilibrium does not impose any constraint on off-equilibrium beliefs, which can potentially lead to multiple equilibria. Therefore, we impose more structure on these beliefs by considering equilibria that satisfy the *D1 criterion* of Cho and Kreps (1987). This criterion first identifies the set of sellers who are more likely to deviate from equilibrium choices. After requiring that buyers have beliefs consistent with this set after observing a deviation, the *D1 criterion* eliminates equilibria in which a seller's payoff from the deviation under the worst buyer's consistent belief is not equilibrium dominated. The following proposition presents our main theoretical result.

***Proposition* 2.** *The balanced-growth path equilibrium is characterized by the following solution to the sequence of problems $\{\mathcal{P}_1(\omega), \ldots, \mathcal{P}_{N_a}(\omega)\}$ for all $\omega \in \Omega$, which is constructed recursively:*

(i) The seller of the lowest unobserved quality a_1 chooses the full-information strategy $\hat{a}(\omega, a_1) = a_1$, $q(\omega, a_1) = q^{FI}(\omega, a_1)$ and $\theta(\omega, \hat{a}(\omega, a_1), q(\omega, a_1)) = \theta^{FI}(\omega, a_1)$, which is characterized by

$$q^{FI}(\omega, a_1) = \nu^{\mathrm{b}}(\omega, a_1) - \frac{\chi}{\mu(\theta^{FI}(\omega, a_1))}$$

and

$$p'(\theta^{FI}(\omega, a_1)) \left(\nu^{\mathrm{b}}(\omega, a_1) - (\beta\nu^{\mathrm{s}}(\omega, a_1) - \delta\omega a_1)\right) = \chi,$$

where $\chi \equiv w_t/\gamma^t$.

(ii) The seller of any unobserved quality $a_k > a_1$ signals his true quality—i.e., $\hat{a}(\omega, a_k) = a_k$. Regarding the terms of trade, there are two cases to consider:

(a) If none of the constraints (10) evaluated at all $a_l < a_k$ bind, then the seller of quality a_k chooses the full-information terms of trade—i.e., $q(\omega, a_k) = q^{FI}(\omega, a_k)$ and $\theta(\omega, \hat{a}(\omega, a_k), q(\omega, a_k)) = \theta^{FI}(\omega, a_k)$.

(b) If at least one of the constraints (10) binds for $l \leq k-1$, then let $\underline{\theta}_l^k$ and $\bar{\theta}_l^k$ denote the lowest and largest solutions θ, respectively, to

$$\nu^{\mathrm{s}}(\omega, a_l) = p\left(\theta\right) \left((1-\psi)q(\omega, a_k) + \psi q^P(\omega, a_l, \hat{a}(\omega, a_k), q)\right) + (1 - p(\theta)) \left(\beta\nu^{\mathrm{s}}(\omega, a_l) - \delta\omega a_l\right),$$

where $q(\omega, a_k) = \nu^{\mathrm{b}}(\omega, a_k) - \frac{\chi}{\mu(\theta)}$ (set $\underline{\theta}_l^k = -\infty$ or $\bar{\theta}_l^k = \infty$ if there is none). Finally, let

$$\underline{\theta}_m^k \text{ with } m = argmin \left\{\underline{\theta}_j^k,\ j \in [1, k-1]\right\}.$$

[7] In our analysis, we omit unreasonable separating equilibria in which sellers of different unobserved qualities choose to set different prices (so buyers can identify their different qualities) and announce different, but untrue, qualities. These equilibria can exist in the case where the inspection-adjusted price does not depend on whether the announced quality is different from the true quality.

Then, $\theta(\omega, a_k) = \underline{\theta}_m^k$ is the unique optimal market tightness if $\beta\nu^{\mathrm{s}}(\omega, a_k) - \delta\omega a_k \geq \beta\nu^{\mathrm{s}}(\omega, a_l) - \delta\omega a_l$ for all $l < k$.

Additionally, the optimal market tightness is lower than under the full-information terms of trade, ie $\underline{\theta}_m^k < \theta^{FI}(\omega, a_k)$

Then, this solution to the sequence of problems $\{\mathcal{P}_1(\omega), \ldots, \mathcal{P}_{N_a}(\omega)\}$ for all $\omega \in \Omega$ is the unique fully revealing separating equilibrium that satisfies the D1 criterion. Finally, there are no pooling equilibria.

Proof. See Appendix A.1.

■

Excluding the seller of the lowest unobserved quality who is never affected by asymmetry of information, Proposition 2 describes two distinct situations. In the first case, sellers are able to choose the unconstrained optimum of their objective as no other seller wants to mimic them when they adopt this strategy. Formally, constraints (10) drop out and the optimal terms of trade are characterized by the first-order condition and the buyer's indifference condition (9). We denote this unconstrained solution as the full-information terms of trade. As we will show below, this situation happens when the signal is informative enough—i.e., when ψ is high enough. The second case arises when at least one other seller wants to mimic at the unconstrained solution, which is formally characterized by at least one of the constraints being violated. Intuitively, a relatively uninformative inspection facilitates mimicking by sellers of lower unobserved qualities. Sellers of high unobserved quality must then adapt their strategy to dicentivize mimicking by lower types and thereby signal their true quality. Thus, the optimal terms of trade become distorted relative to the full-information case. We show that if the seller's values are increasing in the unobserved quality (which is always true for realistically low depreciation rates), then the optimal signaling strategy consists in choosing a lower tightness, and a higher price, than under full information so that the tightest constraint is just binding. This forms the unique fully revealing separating equilibrium; no pooling equilibria exist.

Next, we provide an intuition of the mechanisms through which asymmetric information distorts the equilibrium allocation. For pedagogical purposes, we focus on the case with $\Omega = \{\omega_L, \omega_H\}$ and $\mathcal{A} = \{a_L, a_H\}$ (where L and H denote low and high values, respectively). We also assume that depreciation costs are small relative to the values of sellers—i.e., $\delta \to 0$—which is a reasonable approximation given our calibration strategy below. We first show how the terms of trade change in the cross-section of observed characteristics. We then describe how asymmetric information affects the trade of units with different unobserved characteristics.

Observed characteristics We first consider the case in which trading occurs with full information. In this case, the set of constraints (10) are not binding and the solution to the seller's problem is characterized by the first-order condition

$$p'(\theta(\omega,a))\left(\nu^{\mathrm{b}}(\omega,a)-\beta\nu^{\mathrm{s}}(\omega,a)\right)=\chi$$

for all $(\omega,a)\in\Omega\times\mathcal{A}$, where we have replaced the price $q(\omega,a)$ from the optimal search strategy of the buyer. The optimal choice of market tightness balances the marginal benefit of a higher trading probability (left-hand side) with the reduction in the price required by potential buyers in order to visit the chosen submarket (right-hand side). The following proposition formalizes this result by deriving the optimal price of capital and market tightness for each type of capital under full information.

***Proposition* 3.** *Under full information, the price and market tightness for capital of quality* (ω,a) *are given by*

$$q^{FI}(\omega,a)=\eta\nu^{\mathrm{b}}(\omega,a)+(1-\eta)\beta\nu^{\mathrm{s}}(\omega,a)$$

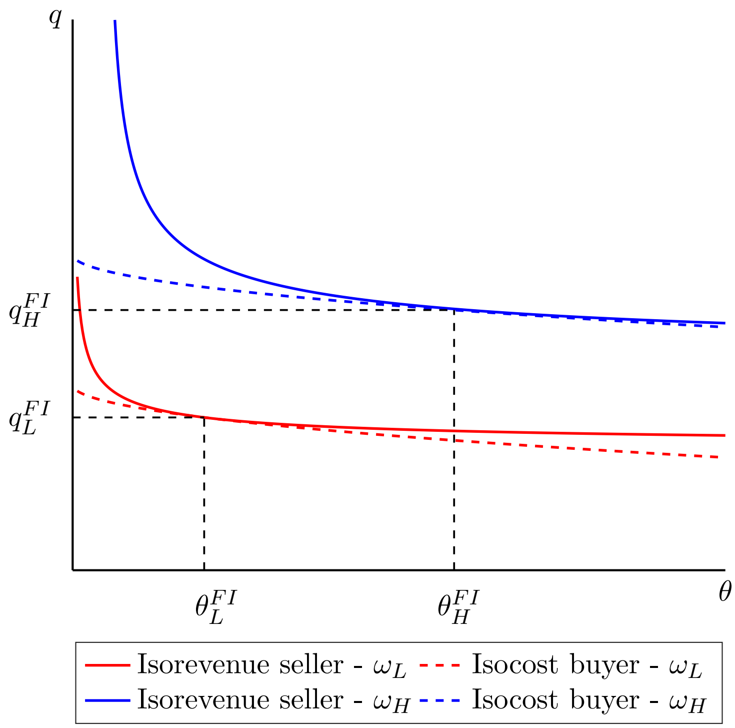
and

$$\theta^{FI}(\omega,a)=\left(\frac{\bar{m}(1-\eta)}{\chi}\left(\nu^{\mathrm{b}}(\omega,a)-\beta\nu^{\mathrm{s}}(\omega,a)\right)\right)^{1/\eta}.$$

Proposition 3 shows that the equilibrium price is a weighted average of the seller's and buyer's value of capital and the selling probability is an increasing function of the surplus $\nu^{\mathrm{b}}(\omega,a)-\beta\nu^{\mathrm{s}}(\omega,a)$. This optimal choice is graphically represented in Figure 1 for types (ω_L,a_L) and (ω_H,a_L) and given search cost χ. Dashed lines represent the isocost curves of buyers, with the highest one corresponding to the high quality ω_H. These curves denote the combination of prices and purchase probabilities that generate the same expected cost to buyers and comes from equation (9). Curves are downward sloping because buyers are indifferent between submarkets if higher prices are associated with higher matching rates with sellers. They are increasing in ω because buyers can obtain higher revenues by using capital of higher quality.

Similarly, solid lines denote the isorevenue curve of sellers—i.e., the combination of prices and market tightness that produce same expected revenues. These are downward sloping because the seller is willing to accept a lower price if the sale probability increases. Note that isorevenue curves have a lower slope for high-quality capital. This results from the outside option (i.e., the continuation value) of the seller being increasing in the quality of its capital, which causes them to require a lower "compensation" in terms of a higher sale probability for a given reduction in the price. In equilibrium, sellers choose the submarket that maximizes their utility subject to the buyers' indifference curves.

Figure 1: Competitive Equilibrium under Full Information



Proposition 3 and Figure 1 show that under full information, the price of a unit of capital and its matching rate are increasing in the quality of capital, which implies the following result.

***Corollary* 1.** *Under full information, capital units with higher prices have higher matching rates:* $q^{\mathrm{FI}}(\omega_H, a) > q^{\mathrm{FI}}(\omega_L, a)$ *and* $p\left(\theta^{\mathrm{FI}}(\omega_H, a)\right) > p\left(\theta^{\mathrm{FI}}(\omega_L, a)\right)$.

To understand the intuition behind this corollary, replace the equilibrium price of capital in the optimal search strategy of the buyer to obtain

$$(1-\eta)\Big(\nu^{\mathrm{b}}(\omega, a) - \beta\nu^{\mathrm{s}}(\omega, a)\Big) = \frac{\theta(\omega, a)\chi}{p\left(\theta(\omega, a)\right)}. \tag{11}$$

Equation (11) requires that in equilibrium, the seller's net benefit from buying a unit of capital must be equal to its expected search cost. As in standard models of directed search, the surplus (given by $\nu^{\mathrm{b}}(\omega, a) - \beta\nu^{\mathrm{s}}(\omega, a)$) is "split" according to the elasticity of the matching function. Thus, since the price of capital scales with the seller's value less than proportionally ($\eta < 1$), the net gain of buying capital is increasing in this value. By non-arbitrage, the expected search cost must be higher for capital units with higher quality—and thus higher value—which implies that buyers (sellers) of these units match at a lower (higher) rate.

Unobserved characteristics Next, we consider the solution to the seller's problem under asymmetric information. As previously shown, capital of the lowest unobserved quality a_L is sold under the full-information terms of trade. However, the choice of the seller of quality a_H capital might be affected by information frictions. In this case, the solution to the seller's problem is characterized by the first-order condition

$$p'(\theta(\omega, a_H)) \left(\nu^{\mathrm{b}}(\omega, a_H) - \beta\nu^{\mathrm{s}}(\omega, a_H)\right) = \chi + \zeta(\omega, a_H) \tag{12}$$

and the complementary slackness condition

$$\begin{aligned} &\zeta(\omega, a_H)\Big[p(\theta^{FI}(\omega, a_L))\left(q^{FI}(\omega, a_L) - \beta\nu^{\mathrm{s}}(\omega, a_L)\right) \\ &- p(\theta(\omega, a_H))\left((1-\psi)q(\omega, a_H) + \psi q^P(\omega, a_H, a_L, q) - \beta\nu^{\mathrm{s}}(\omega, a_L)\right)\Big] = 0 \end{aligned} \tag{13}$$

for all $\omega \in \Omega$, where $\zeta(\omega, a_H)$ denotes the Lagrange multiplier of the no-mimicking constraint, which requires that the lower type a_L does not want to mimic the choices made by the higher type a_H. Notice that the constraint incorporates the fact that sellers who mimic the choices of sellers with other qualities sell at the posted price only when the inspection is uninformative. The presence of the signal in the inspection stage introduces a small deviation from the standard signaling model à la Spence (1973). For high values of ψ—i.e., when the extent of asymmetric information is not severe—sellers do not need to signal their quality, since the probability of detection is high. This intuition is formalized in the following proposition, which is special case of Proposition 2 for the two-type example considered here.

***Proposition* 4.** *Let $\psi^* \in [0, 1]$ be defined as*

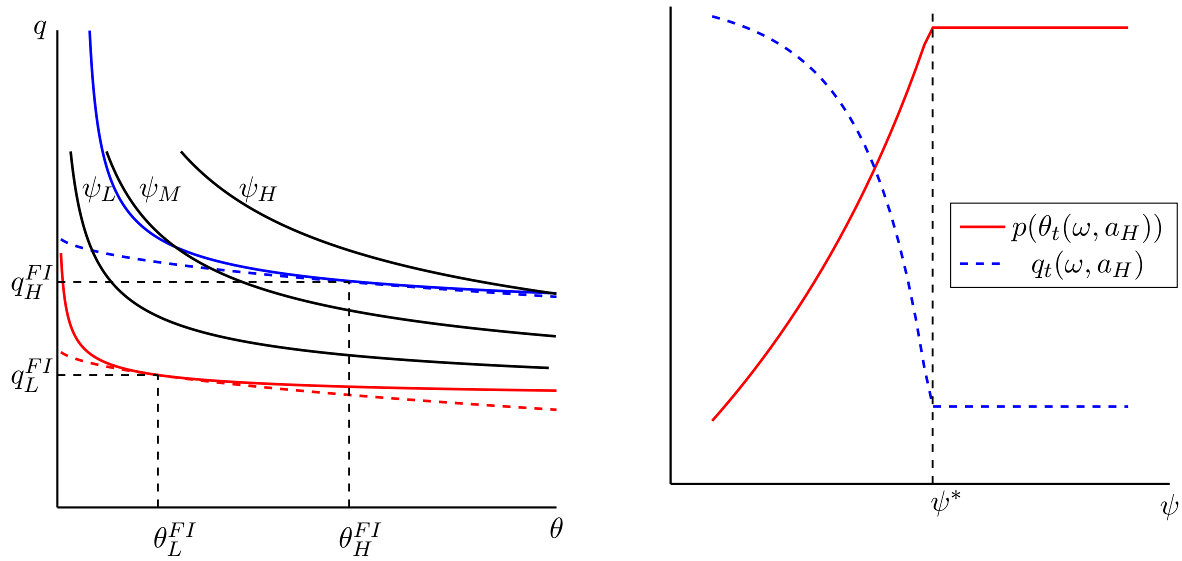
$$\begin{aligned} &p(\theta^{FI}(\omega, a_L))\left[q^{FI}(\omega, a_L) - \beta\nu^{\mathrm{S},FI}(\omega, a_L) + \delta\omega a_L\right] \\ &\qquad = p(\theta^{FI}(\omega, a_H))\left[(1-\psi^*)q^{FI}(\omega, a_H) + \psi^* q^P(\omega, a_H, a_L, q) - \beta\nu^{\mathrm{S},FI}(\omega, a_L) + \delta\omega a_L\right] \end{aligned}$$

Then, for a given ω, the seller of quality a_L chooses the same terms of trade as under full information. For sellers of quality a_H, there are two cases:

(i) $\psi \geq \psi^*$: the incentive-compatibility constraint is not binding (i.e., $\zeta(\omega, a_H) = 0$) and $\theta(\omega, a_H)$ solves the optimality condition (12).

(ii) $\psi < \psi^*$: the incentive compatibility constraint is binding (i.e., $\zeta(\omega, a_H) > 0$) and $\theta(\omega, a_H)$ solves (13). Optimal terms of trade satisfy $q(\omega, a_H) > q^{FI}(\omega, a_H)$ and $p(\theta(\omega, a_H)) < p(\theta^{FI}(\omega, a_H))$.

Figure 2: Competitive Equilibrium under Asymmetric Information



Furthemore, the difference in the expected time to sell across qualities increases as ψ decreases:

$$\left.\frac{d\left[\frac{p(\theta^{FI}(\omega,a_L))}{p(\theta(\omega,a_H))}\right]}{d\psi}\right|_{\psi<\psi^*} < 0. \tag{14}$$

Thus, if information asymmetries are strong enough (i.e., ψ is low enough), then $p(\theta(\omega, a_H)) < p^{FI}(\theta(\omega, a_L))$.

The intuition behind this result can be seen in Figure 2 for the case of a fixed ω and $\mathcal{A} = \{a_L, a_H\}$. In a fully revealing separating equilibrium with signaling, the outcome in the submarket for the lowest quality capital is the same as the one obtained under full information (see Figure 1). However, the outcome in the submarket for high-quality capital could be distorted by the fact that sellers maximize the expected value subject to the constraint that low-quality sellers do not have a strict preference for participating in the same submarket. The different possibilities are illustrated in Panel (a) of Figure 2. In addition to the isorevenue and isocost curves shown in Figure 1, the figure includes the no-mimicking constraint behind the complementary slackness condition (13) for three values of ψ: $\psi_L < \psi_M < \psi^* < \psi_H$. For a given price, any market tightness to the right of the solid black lines violates the constraint.

When the inspection technology is good enough (e.g., ψ_H in Figure 2), the seller of high-quality capital can choose the full-information market tightness. Sellers of low-quality capital do not want to mimic this choice because with a high probability, the inspection reveals their lower quality and they end up selling at a lower price. As ψ decreases below ψ^* (e.g., ψ_M in Figure 2), sellers of low-quality capital are more likely to be able to sell without being detected by the inspection. Then, the full-information tightness violates the constraint and the seller chooses a lower tightness to signal the higher quality of capital. The optimal tightness is determined by the intersection between the no-mimicking constraint and the buyer's isocost curve evaluated at a_H. This lower sale probability is more costly for low-quality sellers given the inspection technology, which could reveal their true type and lead to a low sale price. Because of these additional delays, low-quality sellers weakly prefer their own submarket. If the informativeness of the inspection is very low (e.g., ψ_L in Figure 2), then the required signaling in the form of delays is such that capital of higher quality ends up selling at a lower rate than low-quality capital.

Therefore, ψ is the probability that a sale occurs under asymmetric information and, thus, a sufficient statistic for the potential distortions to terms of trade from information asymmetries (see Panel (b) of Figure 2). The results obtained when $\psi < \psi^*$ were originally established by Guerrieri et al. (2010) in a search model with adverse selection under a different equilibrium selection mechanism. Here, we generalize the theoretical framework to allow for different degrees of information asymmetries, which allows us to identify information frictions in the data.

2.5 Distortions associated with asymmetric information

In this section, we focus on the aggregate economy to describe the alternative channels through which asymmetric information can affect output. Let Y_t, L_t, and $\mathcal{K}_t$ denote gross ouput, aggregate labor used in production, and aggregate capital. We can then write aggregate output as

$$Y_t = \left(\gamma^t L_t\right)^{1-\alpha} \mathcal{K}_t^{\alpha},$$

where $L_t \equiv \bar{h}\gamma_n^t - \int\int \sum_\omega \sum_{\hat{a}} v_{jt}(\omega, \hat{a}, q)\, \mathrm{d}q\, \mathrm{d}j$ and $\mathcal{K}_t \equiv \int \mathcal{K}_{jt}\, \mathrm{d}j$. In turn, we can rewrite the aggregate stock of capital used in production to reflect its unemployment rate and its quality composition:

$$Y_t = \left(\gamma^t L_t\right)^{1-\alpha} \left(\left[\sum_{\omega\in\omega} \sum_{a\in\mathcal{A}} K_t(\omega, a) \right] \left[\mathbb{E}\left(\omega a\right)\left(1 - \mathbb{E}\left(u_t(\omega, a)\right)\right) - \mathbb{C}ov\left(\omega a, u_t(\omega, a)\right) \right] \right)^{\alpha},$$

where $u_t(\omega, a)$ and $K_t(\omega, a)$ denote the aggregate unemployment rate and the aggregate stock of capital of type (ω, a), respectively. Aggregate efficiency units of capital used in production is determined by three components. The first term counts the aggregate units of capital of all types, both employed and unemployed. The second term converts this count into efficiency units used in production by multiplying it by the average efficiency units of capital and the average employment rate of capital. The third term captures the covariance between the type-specific unemployment rate and the efficiency units of capital. Thus, aggregate capital input used in production is high when (i) the number of units of capital is high, (ii) the average efficiency units is high and the average unemployment rate is low, and (iii) unemployed capital tends to be of lower quality.

Given this characterization of aggregate output, we now introduce wedges to highlight the deviation of the equilibrium with asymmetric information from the full-information allocation.

$$Y_t = \left(\gamma^t L_t^{FI}\right)^{1-\alpha} \underbrace{(1+\tau_l)}_{\text{Labor wedge}} \left(\left[\sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} K_t^{FI}(\omega, a) \right] \underbrace{(1+\tau_i)}_{\text{Investment wedge}} \right. \tag{15}$$

$$\left. \left[\mathbb{E}\left(\omega a\right) \left(1 - \mathbb{E}^{FI}\left(u_t(\omega, a)\right) \underbrace{(1+\tau_u)}_{\text{Unemp. wedge}} \right) - \mathbb{C}ov^{FI}\left(u_t(\omega, a), \omega a\right) \underbrace{(1+\tau_{cov})}_{\text{Composition wedge}} \right] \right)^{\alpha}. \tag{16}$$

There are four channels through which asymmetric information can affect aggregate output. First, by distorting the terms of trade, asymmetric information affects the amount of labor used to post vacancies, which directly affects the amount of labor left for production. Second, by affecting the same prices and trading probabilities, asymmetric information distorts the return to capital investment, which reduces the aggregate units of capital in the economy. Finally, asymmetric information affects the aggregate utilization rate of capital in two ways, since it distorts both the average unemployment rate of capital and the composition of the pool of unemployed capital.

3 Measurement

This section presents empirical evidence aimed at measuring the degree of asymmetric information in our framework. Section 3.1 describes the data. Section 3.2 presents key empirical moments linked to the model predictions. Section 3.3 discuss alternative interpretations of these data moments.

3.1 Data

Our data consist of a panel of nonresidential structures (retail and office space) listed for sale and rent. The source of these data is Idealista, one of Europe's leading online real estate intermediaries.[8] The frequency of the panel is monthly, and it includes the universe of capital units that were listed on this platform between 2005 and 2018. The data contain information during the period of time each listing was active online. The dataset includes roughly 8.9 million observations for Spain, where an observation corresponds to a property–month pair. Overall, these observations come from over 1.15 million different capital units.

[8] Idealista is the leading online platform in the real estate market in Spain (see Comparison of users and Comparison of platform). Other papers in the literature have made use of data from online platforms in the real estate market (see, for example, (Piazzesi et al., 2020)).

For each property, we observe a wide range of characteristics that we link to its price. In particular, we observe the address of the property, its construction year, its area, the number of rooms, and whether the property has heat or air conditioning. Table 1 presents some descriptive statistics on these characteristics.[9] The table includes four columns, which are the mean and standard deviation for properties listed for rent and sale, respectively. Our main variable of interest for each capital unit is its price, which we observe for each property at a monthly frequency. The average sale price per square foot is \$162 (expressed in constant 2017 dollars), and monthly rents are around \$1 per square foot per month. The properties are relatively old, with the average age around 26 years. The properties have similar sizes regardless of the operation.

TABLE 1: Descriptive Statistics

	Mean Rent	Std. Rent	Mean Sale	Std. Sale
Price	1.17	6.61	162.27	131.71
Duration	8.55	9.56	10.47	11.21
Construction Date	1986.91	19.43	1987.63	19.50
Area	2669.41	4150.82	3008.89	4619.22
New	0.00	0.06	0.05	0.22
Needs Restoration	0.08	0.27	0.14	0.35
Good Condition	0.91	0.28	0.81	0.40
Rooms	2.57	2.77	2.31	2.99
Restrooms	1.33	1.30	1.21	1.54
Heating	0.38	0.49	0.27	0.45
AC	0.75	0.43	0.64	0.48
E-Mails	2.75	2.12	2.75	2.12
Views	1383.98	2199.22	799.91	1273.95
Clicks	69.74	95.38	44.28	59.08
Number of Obs.	6.7e+05	6.7e+05	4.4e+05	4.4e+05

Note: Price is the price per square foot in constant 2017 dollars. Duration is the number of months a property lasted in the database. Construction date is the year in which the property was built. Property area is measured in square feet. "New" is a categorical variable that takes the value of 1 if the property is new. "Needs Restoration" is a categorical value that takes the value of 1 if the owner declares the property needs reparations. "Good Condition" takes the value of 1 if the property does not need reparations. "Rooms" is the number of separate rooms the property has, similar for "Restrooms". Heating and AC are categorical variables that take the value of 1 when the property has some heating and air conditioning technologies. Emails is the number of times per month a property received an email from a potential customer. Views is the number of times a property appeared in the screen of a potential customer per month. Clicks is the number of times per month a potential customer clicked on the property listing to see its details.

For each capital unit, we also observe three key variables related to its time-to-sell and the attention it receives on the platform. First, based on the identifier of each property, we compute the number of months the unit is listed on the platform, which we refer to as *duration*. Table 1 shows that units for rent and sale remain on the platform 8.6 and 10.5 months on average, respectively. Second,

[9] For those variables that change over time, we first take the average of the variable for each listing and report the average of that variable across listings.

we compute each capital unit's search volume in each month, measured by the number of views and clicks each listing received and the number of emails the seller receives from potential buyers through the platform. Each listing for rent and sale was on average viewed 1,384 and 800 times, respectively. Similarly, listings for rent and sale received 70 and 44 clicks per month, respectively, and 2.75 emails per month.

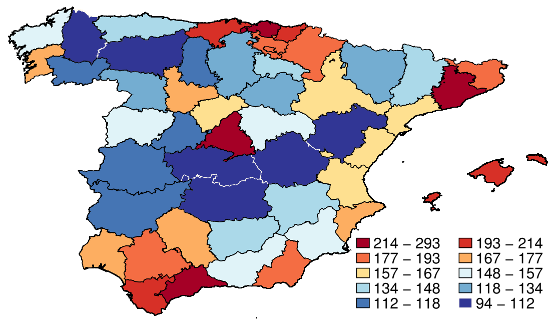
Appendix B provides more details on the data. In particular, Section B.1 describes how the online platform works. Section B.2 studies the representativeness of the dataset, showing that the data from the online platform are consistent with aggregate patterns observed in Spain during the period of analysis in terms of the aggregate evolution of prices and the timing of sales.

Variation over Time and Space We now describe the coverage of the data and the observed variation in prices of listed capital. An important observable dimension of capital prices is the time dimension. Figure 4 shows that, as is well known, the price of capital units experienced large fluctuations over the last 15 years during the boom and bust in the real estate market in Spain. In the markets for both sale and rent, prices declined by more than 50% from the peak in 2007 to the trough in 2012. Since then, prices have remained stable. Another key observable dimension that explains differences in capital prices is location. Figure 3 shows remarkable differences in sale prices across regions at different levels of aggregation. Panel (a) shows the average price across the 50 provinces in Spain; Panel (b) zooms in on the province of Madrid and shows the average price across municipalities in that province; Panel (c) zooms in on the city of Madrid and shows the average price across neighborhoods in the city. These maps demonstrate that locations vary significantly in their capital prices. Finally, Figure 9 in Appendix B.3 shows the evolution of average time to sell over time, which was around 10 months during the boom of the real estate market and then increased to more than 20 months during the subsequent bust.

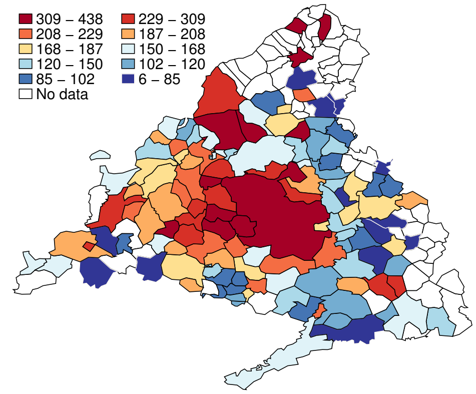
Discussion of the Data The dataset has many advantages for our measurement of asymmetric information. First, it contains panel data for a large amount of nonresidential real estate, with wide geographical and temporal coverage. Second, it contains information on the duration of a listing online. Third, it provides information about the search behavior of potential buyers (monthly number of clicks and emails received). However, the dataset does not contain information on transacted prices. We believe that this should not be a concern for various reasons. First, Figure 6 in the Appendix compares indices of *listed* prices from Idealista with indices of *transacted* prices from the National Registry of Property in Spain. We show that the indices have similar patterns.[10] Second,

[10] Our index leads the index of transacted prices. This is expected since our index consists of listed prices and it will take properties some months to exit the database, be registered as sales and recorded in national statistics.

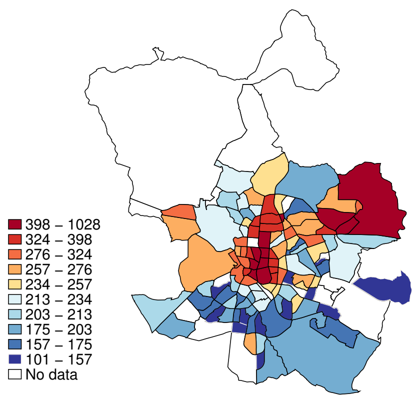
Figure 3: Capital Prices Across Locations



(a) Spain



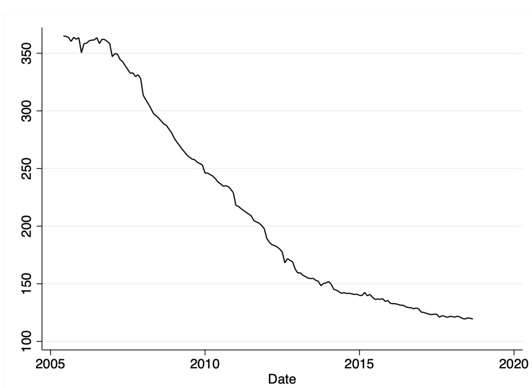
(b) Province of Madrid



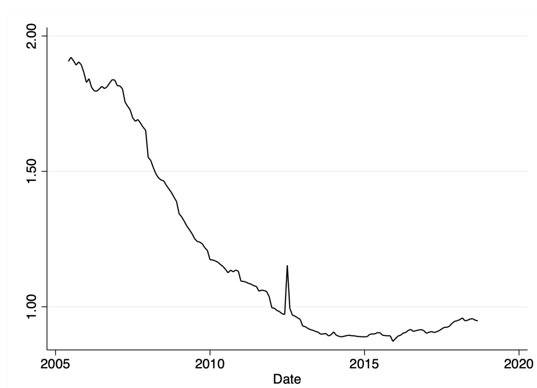
(c) City of Madrid

Note: Each map shows average prices by location expressed in constant 2017 dollars per square foot. The top panel shows average prices across provinces in Spain. The lower-left panel zooms in on the province of Madrid to show substantial heterogeneity across municipalities within this province. The lower-right map shows that, after zooming in on the municipality of Madrid, there is still significant geographical dispersion of prices across neighborhoods.

Figure 4: Evolution of Prices of Capital Units



(a) Capital for Sale



(b) Capital for Rent

Note: The left panel shows the evolution of mean prices at the daily frequency from 2006 to 2017. The right panel shows an equivalent index for rental units. Prices are denominated in constant 2017 dollars per square foot. To compute these price indices, we averaged the prices of all active listings in a given day.

below we show that listed prices are strongly associated with duration on the platform and the attention the listing receives (measured by clicks and emails received). Third, previous papers with access to both listed and transacted prices have shown that the modal property sells at its listed price and that the average property sells within 1.6% of its listed price (see, e.g., Guren, 2018).

Another concern is that duration contains measurement error due to sellers' failure to delete the listing after a sale. This should not be a concern. First, Idealista is a paid service, so it is costly for the seller to keep a listing dormant after the property has been sold. Second, a large fraction of listings are associated with professional sellers (i.e., real estate agents). Finally, to alleviate any remaining concerns, we exploit the fact that the platform asks sellers why they decided to close the listing. Figure 8 in Appendix B.3 compares the histogram of duration for two groups of listings: those that closed the listing because the property was rented out or sold and those that do not provide any explanation. Those histograms are virtually identical.

3.2 Key Data Moments

We now use our data to provide empirical moments linked to the main predictions of the model. We proceed by first measuring the component of a listed priced that can be predicted based on the property's characteristics included in the listing. That is, we estimate hedonic regression to obtain the predicted price based on observable characteristics and the residual price. Then, we analyze how both the predicted and the residual price comove with duration on the market.

3.2.1 Obtaining a Measure of Residual Prices

To quantify the role of observable characteristics of a listing in explaining its price, we estimate the following hedonic pricing model for the (log) price per square foot:

$$log(q_{ilt}) = \nu_{lt} + \gamma X_i + \varepsilon_{ilt} \tag{17}$$

where q_{ijt} is the price (in 2017 dollars) of a capital unit i in location l, listed in month t, ν_{lt} are location and time fixed effects, X_i is a set of observable characteristics included in the listing, and ε_{ilt} is a random error term.[11] Table 2 presents the results of this exercise, showing the R^2 of the regression and the standard deviation of residual prices. To understand how much of the variation in prices is predicted by different groups of characteristics, we estimate multiple regressions including such groups one at a time.

[11] Location fixed effects are defined, for each unit, at the finest geographical level possible in the platform: the neighborhood level in the case of big cities like Madrid or Barcelona, and the city level in smaller cities. Results are similar if we focus only on cities that have available neighborhood information.

Table 2 shows that in the raw data, the standard deviation of prices is 69% and 75% in the market for rent and sale, respectively. Of this variation, time fixed effects can account for 4% and 12% of the variation of prices in each type of transaction. This is perhaps surprising, given the collapse of the real estate market in Spain. However, this result is explained by the fact that in the real estate market, location is a key factor for determining the value of a property. Once we include location and time fixed effects, the R^2 of the regression increases to 45% and 48%, respectively. If we include interactions of the time-location fixed effects with the type of property (office, retail space, or warehouse), area, and age of the property, the R^2 further increases to 73% and 75%. Finally, in the last row of the table, we include the additional characteristics described in Table 1.[12]

A major conclusion from Table 2 is that although the empirical model has a high predictive power for listed prices, between 25% and 30% of the variation in prices is not explained by characteristics included in the listings. Moreover, the standard deviation of the residuals in the benchmark specification, in which we include all available controls, is around 50% of the variation observed in the raw data. Figure 5 shows the distribution of price residuals, which illustrates the relevance of the dispersion in prices not accounted for by the characteristics in the listings. We refer to the dispersion of the residuals from the regression of log prices on all fixed effects and controls of the characteristics in the listings as *residual dispersion.*

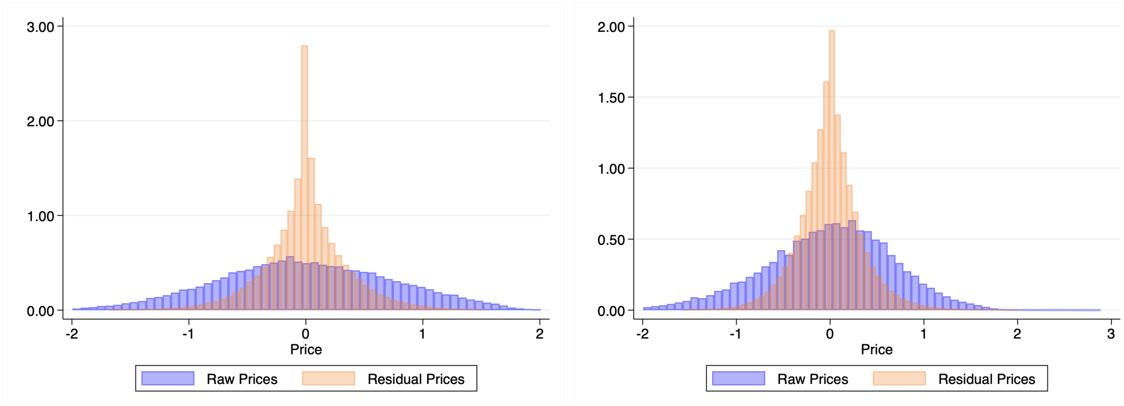
Table 2: Price Variation Accounted for by Listed Characteristics

	Std. Rent	R sq. Rent	Std. Sale	R sq. Sale
Raw data	0.69	0.00	0.75	0.00
Year	0.68	0.04	0.71	0.12
Location	0.51	0.45	0.54	0.48
Year x Loc	0.48	0.51	0.49	0.57
... x Type	0.47	0.54	0.48	0.59
... x Area	0.37	0.71	0.38	0.74
... x Age	0.36	0.73	0.37	0.75
Benchmark	0.36	0.73	0.37	0.75

Note: This table shows the R^2 and standard deviation of residuals of different variations of equation (17). Time and location are fixed effects. Type (office, retail space, or warehouse), area, and age are sets of fixed effects for each of these characteristics. The row labeled Raw Data presents statistics for the demeaned raw log prices. The following rows include the mentioned fixed effects in the regression. The last row includes additional controls for the variables listed in Table 1.

[12] In this analysis, we have focused on the average listed price during the lifetime of the listing. There is another source of price dispersion: price changes during the life of the listing. Table 1 in Appendix B.3 shows that between 5% and 7% of listings change price in a given month. Despite these price changes over time, most of the variation in prices across listings is accounted for by the average price of the listing. Table 2 in Appendix B.3 continues the analysis of Table 2 by further including a listing fixed effect and estimating the regression using the entire panel dataset. Results show that less than 6% of the variation in prices can be accounted for by properties that change their price during the lifetime of the listing.

Figure 5: Distribution of Price Residuals



(a) Price Residuals – Sale (b) Price Residuals – Rent

Note: This figure shows the differences in log prices per square feet with respect to its mean for the raw data and price residuals after including the fixed effects in Table (2). The left panel shows the distributions for sales and the right panel for rentals.

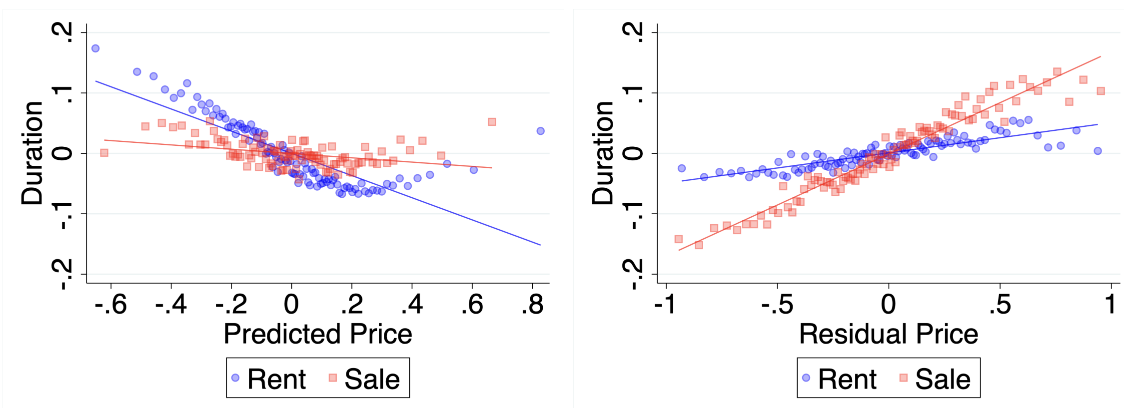
3.2.2 Relationship between Prices and Duration

The last part of the analysis consists of documenting that residual prices and predicted prices have relationships with duration of opposite signs. By *predicted prices*, we refer to the component of a property's price that is linked to its observable characteristics. Since such characteristics are observed in the listing, the theory predicts that, on average, properties with better characteristics (which are reflected by a higher predicted price) should have a shorter duration on the market. By *residual prices*, we refer to the component of a property's price that is *cannot* be explained by its observable characteristics. These residual prices can reflect the quality that is private information of the seller, but can also reflect characteristics that are observed by potential buyers at the inspection stage, but not by the econometrician. A priori, the theory does not offer a prediction about the relation between residual prices and duration. If the inspection technology is good enough, then this relationship should be negative. If instead, the inspection technology is not informative enough, then sellers decide to signal their private information by delaying trade, and we should expect a positive relationship.

Figure 6 plots these relationships. Panel (a) shows that units with higher predicted prices tend to have a shorter duration on the market, which is consistent with model predictions under Full Information. Panel (b) shows that units with higher price residuals tend to have a higher duration, on the market. Table 3 presents the same results in a regression framework. In column (1), we regress (log) duration on (log) prices and obtain a negative and statistically significant relation. If the price of a property increases by 1%, expected duration increases by 0.013%. In the second column, we split the (log) price into two components—predicted and residual prices—and run the same regression. While we obtain a negative and statistically significant relationship between duration and predicted prices, we obtain a positive and statistically significant relationship between duration and residual prices. In the last two columns, we estimate similar regressions, but include location-time-property-type fixed effects, and obtain similar results. [13]

Table 4 and Figure 11 in Appendix B.3 reproduce the same analysis by replacing duration with the average monthly clicks received by a listing (as a proxy for search intensity). Results are consistent with those found for duration. Properties with high predicted prices receive more clicks on average, which is consistent with a shorter duration. On the other hand, properties with high residual prices receive fewer clicks on average, which is consistent with a longer duration. This last set of results is important, because it shows that listed prices do play an important role in attracting or repelling potential buyers by affecting their search behavior.

FIGURE 6: Relationship between Prices and Duration



(A) Predicted Prices (B) Residual Prices

Note: Panel (a) shows the relationship between predicted prices and duration. Panel (b) shows the relationship between residual prices and duration. Price residuals and predicted prices are obtained after running a regression of log prices on a set of fixed effects and observable characteristics. Figures show a binned scatter plot of each relationship, after controlling for location-time-type (offices, retail space, and warehouses) fixed effects.

The different relations that residual and predicted prices have with duration suggest an important role of information. When higher prices stem from listed characteristics, such as the location of the unit—which can be perfectly observed by buyers—they tend to be associated with shorter time to sell. When high prices cannot be easily linked to observable characteristics, then they are associated with longer time to sell. Under the null hypothesis of full information, according to our model, residual prices reflect characteristics of properties not observed by the econometrician. Then, we should expect a negative relation with duration, as is the case with predicted prices. The fact that

[13]The reason behind the inclusion of time-location fixed effects in the regression is to allow for the process of duration on the market to differ over time and location (e.g., the match efficiency could be market specific). However, the theory predicts that if a better observable location contributes positively to the quality of the property, it should also have a positive effect on the trading probability. Therefore, the inclusion of fixed effects is absorbing part of this effect as well.

Table 3: Prices and Duration

	(1) log Dur b/se	(2) log Dur b/se	(3) log Dur b/se	(4) log Dur b/se
log price	0.013***			0.149***
	(0.002)			(0.004)
Predicted Price		-0.018*	-0.073***	
		(0.010)	(0.005)	
Residual Price		0.154***	0.153***	
		(0.004)	(0.004)	
Constant	1.961***	2.108***	2.374***	0.904***
	(0.008)	(0.046)	(0.026)	(0.031)
Observations	456,351	439,680	439,680	445,190
R^2	0.000	0.202	0.010	0.217
Subsample	Sale	Sale	Sale	Sale
Fixed Effects	No	Yes	No	Yes

Note: This table presents the results of a regression of log duration on the two components of prices, residual and predicted prices. The left-hand-side variable is the log duration of a listing, and the right-hand-side variable to be the mean price over the lifetime of the listing. The first column shows a regression of duration on prices. Column 2 regresses duration on predicted prices, residual prices, and location×time×type fixed effects. Columns 3 removes the location fixed effects. Column 4 is a regression of log duration on log prices controlling for all the variables that determine predicted prices. Standard errors are clustered at the location-time level. *, **, and *** represent statistical significance at the 10%, 5%, and 1% level, respectively.

we estimate a positive relation provides evidence that the extent of asymmetric information cannot be zero. This conclusion is more formally supported in the estimation exercise of the model, which allows us to provide a quantitative magnitude of the deviation from full information.

3.3 Alternative interpretations

In this subsection, we discuss alternative theories that could generate a positive relation between residual prices and duration. In each case, we present evidence showing that such alternative explanations are implausible, either because they have trouble reasonably matching the magnitude of the relation between residual prices and duration or because they fail to simultaneously rationalize the relation between predicted prices and duration.

3.3.1 Search theories of price dispersion

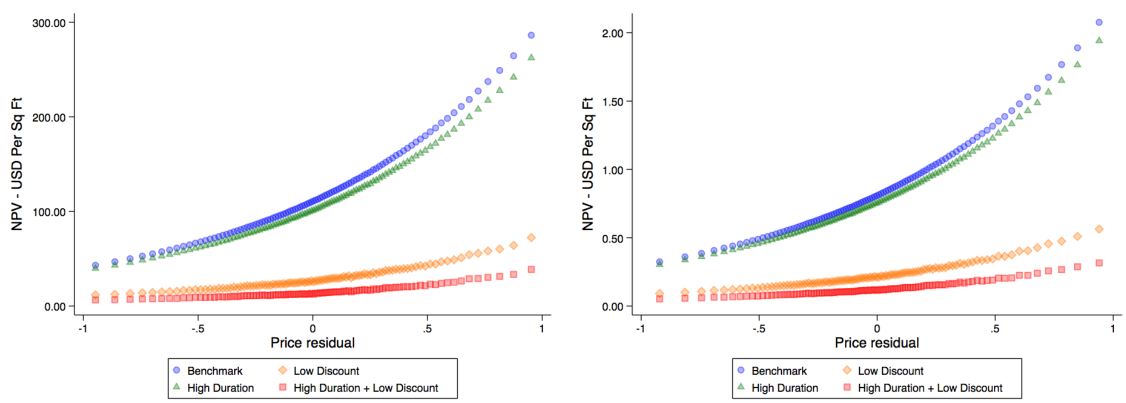
The positive relation between residual price and duration suggests a trade-off between price and time-to-sell, giving rise to a natural explanation from the search literature. If properties and agents are homogeneous, price dispersion obtains from sellers' indifference when choosing the price of their listed units: Higher prices are associated with less search from buyers and more time to sell, and lower prices are associated with more search and shorter time to sell. The key is that the trade-off between prices and time to sell is such that they provide an equivalent expected revenue to the seller. This explanation is akin to that of labor- and product-market models such as those of Burdett and Judd (1983) and Burdett and Mortensen (1998).

To analyze the possibility that such theories explain the positive relation between residual prices and duration, we exploit the data to compute the expected net present values of listed properties at their residual prices and trading probabilities implied by the relation in Panel (b) of Figure 6. That is, we compute

$$\frac{pq}{(1-\beta(1-p))},$$

where q is the residual price, p is the selling probability, and β is the discount factor. The formula is simply a geometric sum corresponding to the expected net present value of listing a price q with a corresponding selling probability p. Note that we abstract from price changes in this calculation and use the mean price instead, since the frequency of price changes is small.[14]

FIGURE 7: Net Present Value of Price-Duration Trade-off



(A) Sale (B) Rent

Figure 7 shows the results for both types of transactions. The blue circle lines correspond to our benchmark calculation, in which we use the observed duration and mean listed prices and a discount factor of $\beta = 0.99$ to compute the net present value. The data show that models of frictional price dispersion cannot explain the positive relationship between residual prices and duration. In the data, the relationship is such that sellers cannot be optimally choosing to randomize: The expected net present value is monotonically increasing in the listed price. Any seller facing such a price-duration trade-off will maximize expected revenue by choosing the highest price we see in the data.

[14]The trading probability in a given month is computed from the duration of each property by $q = 1 - e^{-\lambda}$, where $\lambda = 1/duration$ is the hazard rate.

3.3.2 Heterogeneous sellers

Another potential explanation for the positive relation is that residual prices do not reflect heterogeneity in the properties' unobserved characteristics, but rather heterogeneity in sellers' preferences. Previously, we computed the net present value of a listing for a risk-neutral and relatively patient seller (which strengthens the preference for a higher price). Here, we explore how our previous conclusion is affected by different preferences.

The orange diamonds in Figure 7 correspond to an alternative calculation in which we make sellers extremely impatient ($\beta = 0$). The rationale for this calculation is that we may be ignoring the heterogeneity in sellers' discount factors, which leads us to conclude that some properties have higher returns, when in reality, low-price sellers are setting lower prices in order to sell faster, given their low discount factor. By setting the discount factor to 0, the calculation disproportionally affects properties that have lower trading probabilities and high prices, which flattens the net present value profile. However, even in this extreme scenario, the relation between prices and duration in the data is such that we still find that the net present value is monotonically increasing in the listed price. If, under the preferences of the most impatient seller, a higher residual price with lower selling probability is preferred, then the higher residual price would also be preferred under any other possible discount factor.

The green triangles in Figure 7 show a case in which the realization of duration is worse than the one that is realized in the data. The rationale for this exercise is as follows. If sellers are heterogeneous with respect to their risk aversion, then some sellers may post lower prices to insure themselves. To evaluate the quantitative effect of this argument, we compute the NPV under extreme risk aversion: Sellers form expectations of trading probabilities under a worst-case scenario. We create quantiles of the price residual, and within each quantile we compute the standard deviation of duration across listings. Then, to compute the trading probability of each property, we use the realized duration plus 2 standard deviations of the duration within the quantile to which each property belongs. If the distribution of durations is more dispersed (riskier) for higher prices, then this exercise will shift the net present value of more expensive properties and flatten the NPV profile. The green triangles in Figure 7 show that this is indeed the case, but the quantitative magnitude is small: The NPV profile is still upward sloping. Finally, the red square series combines both sources of seller heterogeneity: It computes the NPV with both a zero discount factor and an adverse duration realizations. Even in this case, the NPV profile is upward sloping.

The conclusion of this analysis is that seller heterogeneity cannot rationalize the positive relationship between residual prices and duration. If it could, the NPV analysis should show that sellers with different preferences should have different NPV-maximizing prices. We find that for a very broad set of preferences (discount factors from 0 to 0.99 and attitude toward risk from risk neutral to extreme forms of risk aversion), all sellers would maximize their expected net present value by choosing the highest price observed in the data.

The last piece of evidence against the role of sellers' preferences is the negative relation between predicted prices and duration we estimate. According to Proposition 3 in our model, sellers with homogeneous properties but different discount factors will optimally choose different prices: The optimal price is increasing in the seller's discount factor, since a higher price is associated with a lower trading probability, which is relatively less costly for more patient sellers. Importantly, this prediction applies in the case with full information. If predicted prices reflect the component of a property's quality that is observable to market participants, then a model with heterogeneous sellers' discount factor should predict a positive relation between predicted prices and duration. However, we instead estimate a negative relationship. Our claim is not that heterogeneity in sellers' preferences is not important, but that such heterogeneity cannot be the main driver of the data.

3.3.3 Heterogeneous holding costs

There is one additional source of sellers' heterogeneity that is not considered by our previous analysis of preference heterogeneity: the presence of heterogeneous holding costs. Sellers must pay these costs until the property is sold, examples are maintenance costs, taxes, debt service costs, etc. If sellers face different costs, then some sellers might be forced to list properties at low prices in order to sell their property faster, as would occur in a fire sale. This would generate the positive relation between residual prices and duration. We explore this possibility by computing the size of this cost that would render sellers indifferent between listing at the highest price without a cost or at their chosen price with a cost. That is, we compute the necessary holding cost of each listing relative to the holding cost of a seller who listed their property at the highest price. We then present the size of this cost as a share of the listed price. The question we seek to answer with this exercise is: How large must the cost be in order to rationalize the choice of a lower residual price?

To compute the (unobserved) cost, c, we solve the following equation:

$$\frac{p_h q_h - c_j(1-p_h)}{1-\beta(1-p_h)} = \frac{p_j q_j - c_j(1-p_j)}{1-\beta(1-p_j)},$$

where q_h and p_h are the price and selling probability of the property with the highest residual price, respectively, and q_j and p_j are the price and probability we observe for property j. In order to rationalize the preference for a lower price and a higher trading probability, the cost must be higher when the difference between prices is larger and when the difference in durations is smaller. We estimate how large these costs must be in order to rationalize the choice of sellers for the case of sales. We present the cost normalized by the price of the most expensive property (in relative terms). Appendix Figure 12 presents the results. We can see that in order for differential holding costs to explain the differences in returns in the data, they must be extremely large. To illustrate, the cost of holding 1 square foot of a property for 1 additional month would have to be larger than the price at which the owner can sell that unit. We conclude that it is unlikely that the bulk of the positive relation between residual prices and duration is explained by the presence of heterogeneous holding costs.

4 Quantitative Analysis

In this section, we combine the model and measurement to quantify the aggregate implications of asymmetric information. Section 4.1 discusses the calibration and Section 4.2 presents counterfactual analyses that vary the degree of asymmetric information.

4.1 Parameterization

We calibrate the model in two steps. First, we exogenously fix a subset of parameters. Second, we calibrate the remaining parameters—which govern the degree of asymmetric information—to match key data moments discussed in Section 3.

Fixed parameters The parameters we fix in the calibration are detailed in Panel (a) of Table 4. A subset of these parameters is shared with the neoclassical stochastic-growth model and is set to standard values from the literature. We set the time unit to a month and $\beta = 0.996$, which is associated with a 4% annual rate of time preference. We assume a period utility $u(c) = \log(c)$ and normalize the labor endowment to $\overline{h} = 1/3$. For firm's technology, we set the share of capital to $\alpha = 0.35$ (consistent with Fernald, 2014) and the growth rate of technical progress to $\gamma = 1.004$, associated with an annual technology growth rate of 1.6%, which is the growth rate per worker the U.S. economy experienced from 1980 to 2015 (data source: BEA). We set the population growth to $\gamma_n = 1.0027$, which is associated with an annual growth rate of the working-age population in the period of analysis of 1% (population aged 15–64, data source: Federal Reserve Bank of St. Louis and OECD).

The second subset of parameters we fix in the calibration is linked to frictions in the decentralized capital market. For the exit rate of firms, we set $\varphi = 0.008$, which corresponds to the 3.2% average exit rate of U.S. establishments, obtained as a weighted average of exit rates for establishments of different sizes reported by the U.S. Census Bureau. For the inspection-adjusted price $q_t^P(.)$ we choose a Nash bargaining protocol parameterized by ϕ (the seller's bargaining power). We set the bargaining parameter to $\phi = 0.5$ as a benchmark in the baseline calibration (used, for example, by Shimer, 2010, in labor markets) and the curvature of the matching function to $\eta = 0.8$, as estimated in Ottonello (2017).

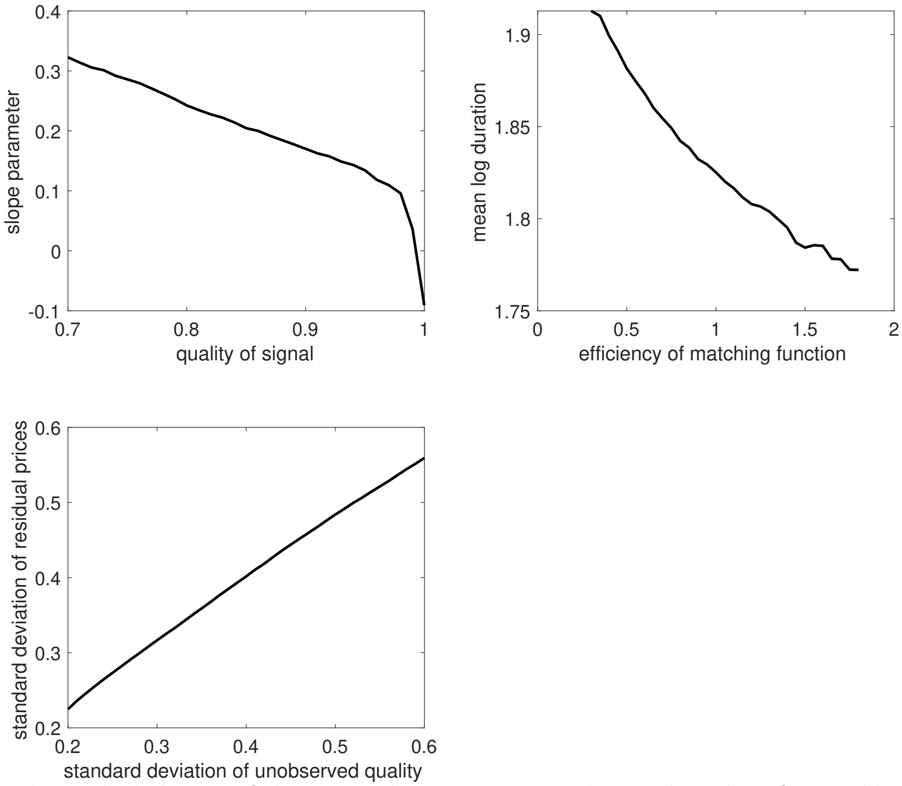
Calibrated parameters The remaining parameters are the most novel parameters in our model governing the extent of information frictions. To parameterize the distributions of observed and unobserved capital qualities, we start from log-normal distributions with standard deviations σ_ω and σ_a, respectively, and discretize the support of its log with 10 evenly spaced points, truncated at -2 and 2 (these values correspond to the range of price found in the data; see Figure 5). The calibrated parameters, listed in Table 4, are σ_ω and σ_a, the matching efficiency $\overline{m}$, and the quality of the inspection technology (or degree of asymmetric information) ψ.

We use the measurement discussed in Section 3 to inform these parameters, targeting the empirical moments reported in Table 5. The calibration strategy of these parameters proceeds as follows. For a given set of parameters, we compute the equilibrium choices of prices and transaction probabilities for each type of capital. Then, we simulate the evolution of multiple units of capital, generate a sample of listed units (similar to that of listed properties in our dataset), and perform the same measurement analysis to obtain these moments in the model-simulated data as we performed with the data in Section 3. Finally, we use a minimum-distance estimator to choose parameter values that match the moments in the data.

Although there is no one-to-one mapping from parameters to moments, we provide intuition regarding the identification of the model parameters. The average selling probability is pinned down by the matching efficiency $\bar{m}$. The standard deviation of capital qualities, σ_ω and σ_a, govern the standard deviation of predicted and residual prices. Given these parameters, the quality of the inspection technology, ψ, governs the covariance between duration and residual prices. We illustrate this in the parameterized model in Figure 8.

Table 5 shows that our parameterized model matches fairly well the moments targeted in our calibration. Additionally, in Table 6 we reproduce the regression results reported in Section 3 between duration and predicted prices and residual prices. There is a positive relationship between residual prices and duration. The regression results also report a negative relationship between predicted prices and duration in both the data and the model. This is an additional test of the quantitative exercise since it was not part of the set of targeted moments.

Figure 8: Identification Illustration



Note: This figure shows the behavior of the targeted moments as we change the value of our calibrated parameters. The top left panel shows the behavior of the slope between log duration and residual prices as a function of the quality of the inspection technology. The top right panel shows the behavior of the average log duration of capital units as we change the efficiency of the matching technology. The bottom left panel shows the behavior of the dispersion of residual prices as a function of the dispersion of unobserved quality. The bottom right panel shows the behavior of the dispersion of predicted prices as a function of the dispersion of observed quality.

Table 4: Parameter Values

Parameter	**Description**	**Value**
A. Fixed parameters		
β	Discount factor	0.9966
α	Share of capital	0.35
γ	Technology growth	1.004
γ_n	Population growth	1.0027
φ	Firms' exit rate	0.0027
ϕ	Bargaining power of seller	0.5
η	Curvature matching technology	0.8
B. Calibrated parameters		
$\bar{m}$	Efficiency matching technology	1.55
σ_ω	SD of observed capital quality	0.65
σ_a	SD of unobserved quality	0.61
ψ	Accuracy inspection technology	0.92

Note: Panel A shows the parameters we fixed throughout our exercise. Panel B shows the parameters that we calibrate by minimizing the distance between four moments in the data and in our simulated model. The four parameters we target are the mean log duration of properties, the standard deviation of residual and predicted prices, and the coefficient of residual prices in a regression of log duration on residual prices and predicted prices.

Table 5: Targeted Moments - Data, Benchmark and Full Information

Moment	Data	Benchmark	FI ($\phi = 0.999$)
Mean Duration	7.55	8.04	1.23
σ predicted prices	0.5934	0.5950	0.6198
σ residual prices	0.5463	0.5626	0.5890
slope $\log dur$ and residual prices	0.154	0.153	- 0.091

Note: This table shows the values of the four targeted moments – mean duration, the standard deviation of predicted prices and residual prices, and the regression coefficient of log duration on residual prices – in the data and in the model when the signal quality takes two values: that of our benchmark calibration and a value arbitrarily close to one.

Table 6: Regression duration on prices - data versus model

	(Data) log(Dur)	(Model) log(Dur)
log(Predicted Price)	-0.018	-0.004
log(Residual Price)	0.154	0.153
Constant	2.11	1.96

Note: This table reproduces the regression coefficients in the data and in the model. Log duration is the left-hand-side variable and we regress it on a constant and our measures of predicted and residual prices. Refer to the empirical section for further details.

4.2 Quantifying the aggregate effects of asymmetric information

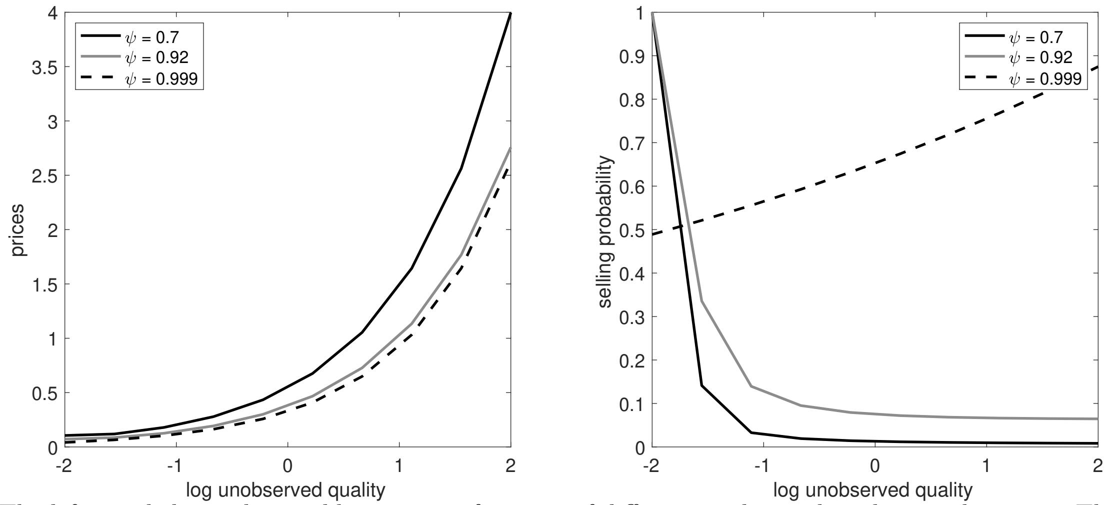
We now examine how the degree of asymmetric information in the economy affects capital-market allocations and macroeconomic variables.

Capital market The solid gray lines of Figure 9 depict the optimal choice of listed prices and associated selling probabilities as a function of capital's unobserved quality. As discussed before, sellers of high-quality capital list them at prices associated with low trading probabilities, which separates them from low-quality sellers.

The solid black lines in Figure 9 show that a lower quality of inspection technology, ψ, is associated with higher prices and lower trading probabilities for high-quality capital. This is because a more imprecise inspection technology creates higher incentives for low-quality capital sellers to mimic higher quality sellers. In turn, high-quality capital sellers respond to the inferior inspection technology by increasing the listed price for their units, which separates them from low-capital quality sellers, who are not willing to bear the cost of the associated low trading probabilities. For this reason, an increase in the degree of asymmetric information driven by a lower capital quality is associated with higher average prices and duration of listed units, particularly at the top of the distribution of capital qualities.

The dashed black lines in Figure 9 show the behavior of prices and selling probabilities when the economy tends to the full-information limit. In that case, prices for high-quality units are lower,

Figure 9: Capital market outcomes for different inspection rates

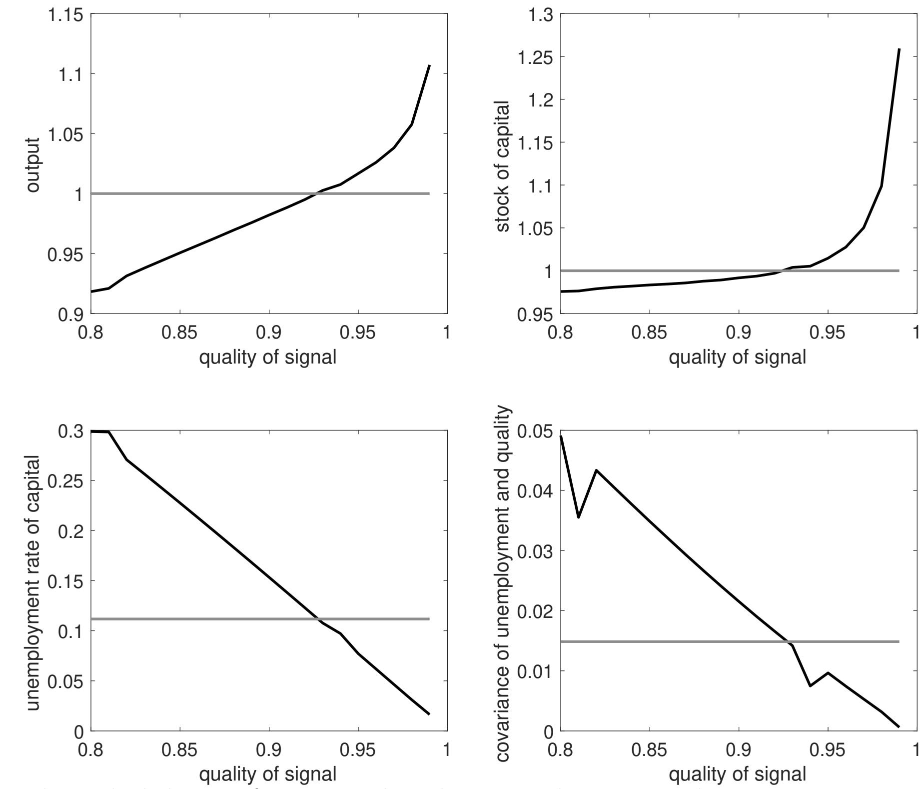


Note: The left panel shows the equilibrium price for units of different unobserved quality on the x-axis. The right panel of the figure shows the selling probabilities for units of different levels of unobserved productivity. In doing this exercise we fix the value of observed productivity, ω, at its median. The three lines correspond to three values of the quality of inspection technology, ψ: a low value $\psi = 0.7$, our calibrated value $\psi = 0.92$, and a large value, $\psi = 0.99$, which captures the equilibrium when the economy tends to the full-information limit.

since the incentives to signal disappear. Selling probabilities are increasing in quality, in a manner analogous to their behavior with respect to observed quality ω. When the inspection technology is sufficiently good, the unobserved quality a can be inferred with enough precision that sellers do not need to signal their quality.

Macroeconomic variables Figure 10 quantifies how the degree of asymmetric information in the economy, driven by changes in the quality of the inspection technology ψ, affects economic activity. Overall, Panel (a) shows that a higher degree of asymmetric information in the economy is associated with a lower steady-state income per capita. Panels (b)-(d) show the three channels discussed in Section 2.5 through which asymmetric information affects economy activity. First, Panel (b) shows that higher information asymmetries lead to a lower capital stock. This is because higher information asymmetries are associated with a lower revenue for sellers of high-quality capital, which decreases the returns to producing capital goods. Second, Panel (c) shows that higher information asymmetries lead to a higher unemployment rate of capital. As information asymmetries increase, so do the listed prices of high-quality capital sellers, which decreases selling probabilities and increases the duration in unemployment of listed units. Third, Panel (d) shows that a higher degree of asymmetric information is associated with a lower quality of employed capital. This is because information asymmetries disproportionally affect the allocation for sellers of high-quality capital, who have to prevent mimicking by lower types through higher prices and lower trading probabilities.

Figure 10: Capital market outcomes for different inspection rates



Note: This figure shows the behavior of output, and its three main drivers, according to our aggregation exercise, as we change the quality of the inspection technology, ψ. The two panels, of output and the stock of capital, are expressed as a ratio of the levels for our calibrated value of $\psi = 0.89$. The gray horizontal lines show the values of each statistic for our calibrated model.

5 Conclusion

In this paper, we show that information asymmetries in capital markets have important macroeconomic implications, affecting an economy's capital stock, misallocation, and income levels. This conclusion emerges from using as a laboratory economy a quantitative general-equilibrium capital-accumulation model with asymmetric information that is consistent with micro level data on the price and duration of individual capital units listed for trade. The results of our paper suggest the importance of studying capital-market policies designed to address potential inefficiencies that arise from information asymmetries. For example, one can use our quantitative framework to investigate the welfare benefits of implementing a pooling equilibrium (e.g., by setting taxes on capital prices that prevent signaling). In addition, the significant macroeconomic gains of reducing asymmetric information suggest that it is of first-order importance to understand agents' incentives of developing inspection technologies that mitigate information frictions. This could be done with empirical analysis that exploits changes in behavior caused by changes in the extent of information frictions, and in a version of the model in which data and information technologies are endogenous (e.g., Jones and Tonetti, 2020; Farboodi and Veldkamp, 2021). We leave this analysis for future research.

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A Theory Appendix

A.1 Proofs of propositions

Proof of Proposition 1

Here we prove a more general version of Proposition 1, in which we endogenize all selling and buying decisions of firms and households. Let us denote by $b^{FI}(\omega, \hat{a}, q, a) \in \{0,1\}$ the decision of buyers to purchase the unit on submarket $(\omega, \hat{a}, q)$ conditional on learning from the inspection that it is of quality a. Similarly, let $b(\omega, \hat{a}, q) \in \{0,1\}$ be the decision of buyers to purchase the unit conditional on visiting the submarket $(\omega, \hat{a}, q)$ and not learning the true quality from the inspection. Let $s(\omega, a)$ be the seller's decision to post the unit of quality (ω, a) for sale. In what follows, we drop the subscript j of an individual firm.

Household's problem The recursive optimization of the household can be written as

$$V_{Ht}(\mathbf{k}) = \max_{\substack{\{v(\omega,\hat{a},q),q(\omega,a),b(\omega,\hat{a},q),\\ b^{FI}(\omega,\hat{a},q,a),s(\omega,a),\hat{a}(\omega,a),\mathbf{k}',c,i\}}} u(c)\gamma_n^t + \beta V_{Ht+1}(\mathbf{k}'),$$

subject to the per-period budget constraint

$$c\gamma_n^t + i + \delta \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} \omega a \left(k'(\omega, a) - k^{\mathrm{s}}(\omega, a)\right) = w_t \bar{h} \gamma_n^t + x^{\mathrm{s}} - x^{\mathrm{b}} + Div_{Ft},$$

the law of motion of capital of quality (ω, a)

$$k'(\omega, a) = k^{\mathrm{b}}(\omega, a) - k^{\mathrm{s}}(\omega, a) + k(\omega, a) + ig(\omega, a) + \varphi K_{Ft}(\omega, a),$$

where total purchases of quality (ω, a) are given by

$$k^{\mathrm{b}}(\omega, a) = \sum_{\hat{a}} \int_{q \in \mathbb{R}_+} \pi_t(a|\omega, \hat{a}, q)[\psi b^{FI}(\omega, \hat{a}, q, a) + (1-\psi) b(\omega, \hat{a}, q)] \mu_t\left(\theta\left(\omega, \hat{a}, q\right)\right) v(\omega, \hat{a}, q)\, \mathrm{d}q,$$

and total sales are given by

$$k^{\mathrm{s}}(\omega, a) = \left(\psi b_t^{FI}(\omega, \hat{a}(\omega, a), q(\omega, a), a) + (1-\psi) b_t(\omega, \hat{a}(\omega, a), q(\omega, a))\right) p(\theta(\omega, \hat{a}, q(\omega, a)) s(\omega, a) k(\omega, a),$$

total costs of buying capital are given by

$$x^{\mathrm{b}} = \sum_{\hat{a}} \sum_{\omega \in \Omega} \int_{q \in \mathbb{R}_+} \Big[(\psi \sum_{a \in \mathcal{A}} \pi_t(a|\omega, \hat{a}, q) q_t^P(\omega, a, \hat{a}, q) b^{FI}(\omega, q, \hat{a}, a)$$
$$+ (1-\psi) q b(\omega, \hat{a}, q)) \mu_t \left(\theta\left(\omega, \hat{a}, q\right)\right) + w_t \Big] v(\omega, \hat{a}, q) \, \mathrm{d}q,$$

and total revenues from selling capital are given by

$$x^{\mathrm{s}} = \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} \Big[\psi b_t^{FI}(\omega, \hat{a}(\omega, a), q(\omega, a), a) q^P(\omega, a, \hat{a}(\omega, a), q)$$
$$+ (1-\psi) b_t(\omega, \hat{a}(\omega, a), q(\omega, a)) q(\omega, a) \Big] p(\theta(\omega, q(\omega, a))) s(\omega, a) k(\omega, a),$$

and the nonnegativity constraints $v(\omega, \hat{a}, q) \geq 0 \ \forall (\omega, \hat{a}, q) \in \Omega \times \mathcal{A} \times \mathbb{R}_+$. The optimal level of investment $i > 0$ is given by the first-order condition

$$1 = \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} g(\omega, a) \lambda_t(\mathbf{k}) \nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{k}),$$

where $\lambda_t(\mathbf{k}) \equiv \beta \gamma_n \frac{u_{ct+1}(\mathbf{k}_{Ht+1}(\mathbf{k}))}{u_{ct}(\mathbf{k})}$, with $\mathbf{k}_{Ht+1}(\mathbf{k})$ is the matrix of policy function for capital accumulation associated with problem (2); and $\nu_t^{\mathrm{s}}(\omega, a, \mathbf{k}) \equiv \frac{\partial V_{Ht}(\mathbf{k})}{\partial k(\omega, a)} \frac{1}{u_{ct}(\mathbf{k}) \gamma^t}$ is the marginal value of capital of type (ω, a) measured in final goods, which satisfies the recursive problem (its notation anticipates a later result that shows that households only sell capital):

$$\nu_t^{\mathrm{s}}(\omega, a, \mathbf{k}) =$$
$$s(\omega, a) p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a))) \Big[\psi b_t^{FI}(\omega, \hat{a}(\omega, a), q(\omega, a), a) q_t^P(\omega, a, \hat{a}(\omega, a), q) + (1-\psi) b_t(\omega, \hat{a}(\omega, a), q(\omega, a)) q(\omega, a) \Big]$$
$$+ \Big(1 - (\psi b_t^{FI}(\omega, \hat{a}(\omega, a), q(\omega, a), a) + (1-\psi) b_t(\omega, \hat{a}(\omega, a), q(\omega, a))) s(\omega, a) p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a)))\Big) \left[\lambda_t(\mathbf{k}) \nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{k}_{Ht+1}(\mathbf{k})) - \delta \omega a\right].$$

Firm's problem The recursive optimization problem faced by firms can be written as

$$V_{Ft}(\mathbf{k}) = \max_{\substack{\{l, v(\omega, \hat{a}, q), q(\omega, a), b(\omega, \hat{a}, q), \\ b^{FI}(\omega, \hat{a}, q, a), s(\omega, a), \hat{a}(\omega, a), \mathbf{k}'\}}} \mathbb{E}_a [div_t + \Lambda_{t,t+1} \left((1-\varphi) V_{Ft+1}(\mathbf{k}') + \varphi V_{t+1}^{\mathrm{exit}}(\mathbf{k}')\right)],$$

subject to the definition of per-period dividends

$$div_t = \left(\sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} \omega a k'(\omega, a) \right)^{\alpha} (\gamma^t l)^{1-\alpha} - w_t l - \delta \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} \omega a k'(\omega, a) - x_t^b + x_t^s,$$

the law of motion of capital of quality (ω, a)

$$k'(\omega, a) = k_t^{\mathrm{b}}(\omega, a) - k_t^{\mathrm{s}}(\omega, a) + k_t(\omega, a),$$

where total purchases of quality (ω, a) are given by

$$k_t^{\mathrm{b}}(\omega, a) = \sum_{\hat{a}} \int_{q \in \mathbb{R}_+} \pi_t(a|\omega, \hat{a}, q)[\psi b^{FI}(\omega, \hat{a}, q, a) + (1-\psi) b(\omega, \hat{a}, q)] \mu_t\left(\theta\left(\omega, \hat{a}, q\right)\right) v(\omega, \hat{a}, q)\, \mathrm{d}q,$$

and total sales are given by

$$k_t^{\mathrm{s}}(\omega, a) = \left(\psi b_t^{FI}(\omega, \hat{a}(\omega, a), q(\omega, a), a) + (1-\psi) b_t(\omega, \hat{a}(\omega, a), q(\omega, a))\right) p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a)) s(\omega, a) k(\omega, a),$$

total costs of buying capital are given by

$$\begin{aligned} x_t^{\mathrm{b}} = \sum_{\hat{a}} \sum_{\omega \in \Omega} \int_{q \in \mathbb{R}_+} \Big[& (\psi \sum_{a \in \mathcal{A}} \pi_t(a|\omega, \hat{a}, q) q_t^P(\omega, a, \hat{a}, q) b^{FI}(\omega, q, \hat{a}, a) \\ & + (1-\psi) q b(\omega, \hat{a}, q)) \mu_t\left(\theta\left(\omega, \hat{a}, q\right)\right) + w_t \Big] v(\omega, \hat{a}, q)\, \mathrm{d}q, \end{aligned}$$

and total revenues from selling capital are given by

$$\begin{aligned} x_t^{\mathrm{s}} = \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} \Big[& \psi b_t^{FI}(\omega, \hat{a}(\omega, a), q(\omega, a), a) q^P(\omega, a, \hat{a}(\omega, a), q) \\ & + (1-\psi) b_t(\omega, \hat{a}(\omega, a), q(\omega, a)) q(\omega, a) \Big] p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a))) s(\omega, a) k(\omega, a), \end{aligned}$$

and the nonnegativity constraints $v(\omega, \hat{a}, q) \geq 0 \ \forall (\omega, \hat{a}, q) \in \Omega \times \mathcal{A} \times \mathbb{R}_+$.

The recursive problem of the firm features a static choice of labor demand and only depends on the number of efficiency units of capital $\mathcal{K}' = \sum_{\omega \in \Omega} \sum_{a \in \mathcal{A}} \omega a k'(\omega, a)$. The first-order condition with respect to l is given by

$$\mathcal{K}'^{\alpha} \gamma^{t(1-\alpha)} (1-\alpha) l^{-\alpha} = w_t,$$

which can be rewritten as

$$l = \mathcal{K}' \left(\frac{(1-\alpha)\gamma^t}{w_t} \right)^{\frac{1-\alpha}{\alpha}}.$$

Hence, labor demand is linear in $\mathcal{K}'$, which proves the last part of Proposition 1. We can express the revenue from production as

$$\Phi_t(\mathbf{k}') = \mathcal{K}'^{\alpha} (\gamma^t l)^{1-\alpha} - w_t l.$$

Replacing our expression for the optimal labor demand, we obtain that $\Phi_t(\mathbf{k}') = Z_t \mathcal{K}'$, where

$$Z_t \equiv \alpha \left(\frac{\gamma^t (1-\alpha)}{w_t} \right)^{\frac{1-\alpha}{\alpha}} .$$

Given this result, we can now re-express the problem of the firm as

$$V_{Ft}(\mathbf{k}) = \max_{\substack{\{v(\omega,\hat{a},q), q(\omega,a), b(\omega,\hat{a},q),\\ b^{FI}(\omega,\hat{a},q,a), s(\omega,a), \hat{a}(\omega,a), \mathbf{k}'\}}} \mathbb{E}_a \left(Z_t - \delta\right) \mathcal{K}' - x_t^b + x_t^s + \Lambda_{t,t+1}\left((1-\varphi) V_{Ft+1}(\mathbf{k}') + \varphi V_{t+1}^{\text{exit}}(\mathbf{k}')\right),$$

$$k'(\omega,a) = k^{\text{b}}(\omega,a) - k^{\text{s}}(\omega,a) + k(\omega,a),$$

$$k^{\text{b}}(\omega,a) = \sum_{\hat{a}} \int_{q\in\mathbb{R}_+} \pi_t(a|\omega,\hat{a},q)[\psi b^{FI}(\omega,\hat{a},q,a) + (1-\psi) b(\omega,\hat{a},q)]\mu_t\left(\theta\left(\omega,\hat{a},q\right)\right) v(\omega,\hat{a},q)\,\mathrm{d}q,$$

$$k^{\text{s}}(\omega,a) = \left(\psi b_t^{FI}(\omega,\hat{a}(\omega,a),q(\omega,a),a) + (1-\psi) b_t(\omega,\hat{a}(\omega,a),q(\omega,a))\right) p(\theta(\omega,\hat{a}(\omega,a),q(\omega,a)) s(\omega,a) k(\omega,a),$$

$$x_t^{\text{b}} = \sum_{\hat{a}} \sum_{\omega\in\Omega} \int_{q\in\mathbb{R}_+} \Big[(\psi \sum_{a\in\mathcal{A}} \pi_t(a|\omega,\hat{a},q) q_t^P(\omega,a,\hat{a},q) b^{FI}(\omega,q,\hat{a},a)$$

$$+ (1-\psi) q b(\omega,\hat{a},q))\mu_t\left(\theta\left(\omega,\hat{a},q\right)\right) + w_t\Big] v(\omega,\hat{a},q)\,\mathrm{d}q,$$

$$x_t^{\text{s}} = \sum_{\omega\in\Omega} \sum_{a\in\mathcal{A}} \Big[\psi b_t^{FI}(\omega,\hat{a}(\omega,a),q(\omega,a),a) q^P(\omega,a,\hat{a}(\omega,a),q)$$

$$+ (1-\psi) b_t(\omega,\hat{a}(\omega,a),q(\omega,a)) q(\omega,a)\Big] p(\theta(\omega,\hat{a}(\omega,a),q(\omega,a))) s(\omega,a) k(\omega,a),$$

$$v(\omega,\hat{a},q) \geq 0 \;\forall (\omega,\hat{a},q) \in \Omega \times \mathcal{A} \times \mathbb{R}_+.$$

Next, we conjecture that $V_{Ft}(\mathbf{k}) = \sum_{\omega\in\Omega} \sum_{a\in\mathcal{A}} V_t^{\text{b}}(\omega,a) k(\omega,a)$. Let us denote by $\xi_t(\omega,\hat{a},q)$ the Lagrange multiplier associated with the nonnegativity constraint for vacancies in all submarkets $(\omega,\hat{a},q) \in \Omega \times \mathcal{A} \times \mathbb{R}+$. The first-order condition with respect to $v(\omega,\hat{a},q)$ is

$$\mathbb{E}_a[\psi b^{FI}(\omega,\hat{a},q,a) + (1-\psi) b(\omega,\hat{a},q)]\mu_t\left(\theta\left(\omega,\hat{a},q\right)\right)\left((Z_t - \delta)\omega a + \Lambda_{t,t+1}((1-\varphi) V_{t+1}^{\text{b}}(\omega,a) + \varphi \nu_{t+1}^{\text{s}}(\omega,a,\mathbf{K}_{Ht+1}))\right) =$$

$$[(\psi \mathbb{E}_a q_t^P(\omega,a,\hat{a},q) b^{FI}(\omega,q,\hat{a},a) + (1-\psi) q b(\omega,\hat{a},q))\mu_t\left(\theta\left(\omega,\hat{a},q\right)\right) + w_t] + \xi_t(\omega,\hat{a},q),$$

together with the complementary slackness condition $\xi_t(\omega,\hat{a},q) v(\omega,\hat{a},q) = 0$. These conditions do not depend on the firm's individual capital holdings and state that the purchased units of capital are bought at a cost equal to their marginal product. We multiply the first-order condition above by $v(\omega,\hat{a},q)$ and insert it in the objective of the firm, which then becomes

$$V_{Ft}(\mathbf{k}) = \max_{\substack{\{q, b(\omega,q,\hat{a}), b^{FI}(\omega,q,\hat{a},a),\\ s(\omega,a), \hat{a}(\omega,a)\}}} \sum_{\omega\in\Omega} \sum_{a\in\mathcal{A}}$$

$$\Big[(Z_t - \delta)\omega a \left(1 - [\psi b^{FI}(\omega,\hat{a}(\omega,a),q(\omega,a),a) + (1-\psi) b(\omega,\hat{a}(\omega,a),q(\omega,a))] p\left(\theta(\omega,\hat{a}(\omega,a),q(\omega,a))\right) s(\omega,a)\right) k(\omega,a)$$

$$+ [\psi b^{FI}(\omega,\hat{a}(\omega,a),q(\omega,a),a) q_t^P(\omega,a,\hat{a}(\omega,a),q) + (1-\psi) b(\omega,\hat{a}(\omega,a),q(\omega,a)) q(\omega,a)] p\left(\theta(\omega,\hat{a}(\omega,a),q(\omega,a))\right) s(\omega,a) k(\omega,a)$$

$$+ \Lambda_{t,t+1}((1-\varphi)\nu_{t+1}^{\text{b}}(\omega,a) + \varphi \nu_{t+1}^{\text{s}}(\omega,a,\mathbf{K}_{Ht+1}))$$

$$\times \left(1 - [\psi b^{FI}(\omega,\hat{a}(\omega,a),q(\omega,a),a) + (1-\psi) b(\omega,\hat{a}(\omega,a),q(\omega,a))] p\left(\theta(\omega,\hat{a}(\omega,a),q(\omega,a))\right) s(\omega,a)\right) k(\omega,a)\Big],$$

where we have used the fact that the total cost of the new purchased units equals their marginal value, so the terms x^b and k^b cancel each other. This shows the linearity of the firm's value function with respect to $\mathbf{k}$. In what follows, we show that households are the sellers and firms are the buyers in the capital market. We also show that, under certain assumptions about $q_t^P(\omega, a, \hat{a}(\omega, a), q)$, buyers always choose to buy the capital unit after matching with a seller.

Firm's selling decision and household's buying decision Here we show that firms buy capital but do not sell it, and that households sell capital but do not buy it (although they invest to produce capital). For this, recall that the value of a capital unit is symmetric among all firms and households because it does not depend on individual capital holdings.

Consider first the situation in which all firms and households want to either buy or sell. Then markets break down, which cannot be an equilibrium. Alternatively, suppose that households want to buy capital. We see from the expression of the objective of households and firms that the marginal value of a capital unit for firms $\nu_t^{\mathrm{b}}(\omega, a)$ is larger than the marginal value of a capital good for the household $\nu_t^{\mathrm{s}}(\omega, a, \mathbf{K}_{Ht+1})$ as long as $Z_t > 0$. Hence, if households want to buy, then firms want to buy as well. Then markets break down, which cannot be an equilibrium. This implies that we can simplify the problem: Households never buy capital (otherwise there are no sellers), and firms never sell capital as long as $Z_t > 0$. Hence, the optimal household's policy is $s(\omega, a) = 0$, which simplifies the firm's marginal value of capital of quality $k(\omega, a)$ to

$$\nu_t^{\mathrm{b}}(\omega, a) = (Z_t - \delta)\omega a + \Lambda_{t,t+1}\left[(1-\varphi)\nu_{t+1}^{\mathrm{b}}(\omega, a) + \varphi\nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{K}_{Ht+1})\right],$$

which proves the second result of Proposition 1. This result also simplifies the household's marginal value of a capital unit to

$$\begin{aligned}
\nu_t^{\mathrm{s}}(\omega, a, \mathbf{k}) = & \max_{\{q_t(\omega,a), \hat{a}_t(\omega,a)\}} p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a)))\Big[\psi b_t^{FI}(\omega, \hat{a}(\omega, a), q(\omega, a), a) q_t^P(\omega, a, \hat{a}(\omega, a), q) \\
& + (1-\psi) b_t(\omega, \hat{a}(\omega, a), q(\omega, a)) q(\omega, a)\Big] \\
& + \Big(1 - (\psi b_t^{FI}(\omega, \hat{a}(\omega, a), q(\omega, a), a) + (1-\psi) b_t(\omega, \hat{a}(\omega, a), q(\omega, a))) p(\theta(\omega, \hat{a}(\omega, a), q(\omega, a)))\Big) \\
& \times \left[\lambda_t(\mathbf{k})\nu_{t+1}^{\mathrm{s}}(\omega, a, \mathbf{k}_{Ht+1}(\mathbf{k})) - \delta\omega a\right].
\end{aligned}$$

Optimal purchase decision Here, we characterize the optimal purchase decision. There are two cases to consider: Either the inspection is successful and the true type (ω, a) is revealed, or it is unsuccessful and only $(\omega, \hat{a}, q)$ is known. We handle both cases successively.

The firm's first-order condition with respect to $b(\omega, q)$ is

$$\frac{\partial V_{Ft}(\mathbf{k})}{\partial b(\omega,\hat{a},q)} = (1-\psi)\mu_t(\theta(\omega,\hat{a},q)) \left(\sum_{a\in\mathcal{A}} \pi_t(a|\omega,\hat{a},q) \left[(Z_t-\delta)\omega a + \Lambda_{t,t+1}\left[(1-\varphi)\nu^{\mathrm{b}}_{t+1}(\omega,a) + \varphi\nu^{\mathrm{s}}_{t+1}(\omega,a,\mathbf{K}_{Ht+1})\right]\right] - q \right),$$

which can be rewritten as

$$\frac{\partial V_{Ft}(\mathbf{k})}{\partial b(\omega,\hat{a},q)} = (1-\psi)\mu_t(\theta(\omega,\hat{a},q))[\mathbb{E}_a\left(\nu^{\mathrm{b}}_t(\omega,a)|\omega,\hat{a},q\right) - q].$$

Hence, the optimal purchase policy when the inspection is not informative is given by

$$b(\omega,\hat{a},q) = \begin{cases} 1 \text{ if } \mathbb{E}_a\left(\nu^{\mathrm{b}}_t(\omega,a)|\omega,\hat{a},q\right) \geq q \\ 0 \text{ otherwise} \end{cases}.$$

Similarly, the firm's first-order condition with respect to $b^{FI}(\omega,\hat{a},q,a)$ is

$$\frac{\partial V_{Ft}(\mathbf{k})}{\partial b^{FI}(\omega,\hat{a},q,a)} = \psi\mu_t(\theta(\omega,\hat{a},q))\pi_t(a|\omega,\hat{a},q)\left((Z_t-\delta)\omega a + \Lambda_{t,t+1}\left[(1-\varphi)\nu^{\mathrm{b}}_{t+1}(\omega,a) + \varphi\nu^{\mathrm{s}}_{t+1}(\omega,a,\mathbf{K}_{Ht+1})\right] - q^P_t(\omega,\hat{a},a,q)\right),$$

which can be rewritten as

$$\frac{\partial V_{Ft}(\mathbf{k})}{\partial b^{FI}(\omega,\hat{a},q,a)} = \psi\mu_t(\theta(\omega,\hat{a},q))\pi_t(a|\omega,\hat{a},q)[\nu^{\mathrm{b}}_t(\omega,a) - q^P_t(\omega,\hat{a},a,q)].$$

Hence, the optimal purchase policy when the inspection is informative is given by

$$b^{FI}(\omega,\hat{a},q,a) = \begin{cases} 1 \text{ if } \nu^{\mathrm{b}}_t(\omega,a) \geq q^P_t(\omega,\hat{a},a,q) \\ 0 \text{ otherwise} \end{cases}.$$

Since we are interested in settings in which trade occurs after an informative inspection as long as the surplus is positive, we impose the following constraint on the post-inspection price $q^P_t(\omega,\hat{a},a,q)$.

***Assumption* 1.** *The post-inspection price satisfies* $q^P_t(\omega,a,\hat{a},q) \in [\min(q,\nu^{\mathrm{s}}_t(\omega,a,\mathbf{K}_{Ht})), \min(q,\nu^{\mathrm{b}}_t(\omega,a))]$ *for all* $\omega\in\Omega, \hat{a}, a\in\mathcal{A}, q\in\mathbb{R}+, \mathbf{K}_{Ht}\in\mathbb{R}+$.

Given the partial derivative with respect to $b^{FI}(\omega,\hat{a},q,a)$ derived above, this assumption ensures that a transaction always happens after an informative inspection.

■

Proof of Propositions 2 and 3

Here we provide a proof for both Propositions 2 and 3, as Proposition 3 is a special case of Proposition 2 when $N_a = 2$. The proof is split into eight distinct steps that we list below.

Step 1: We lay out the implications of the D1 criterion in our problem and show that the price in a given submarket is increasing in the average expected quality.

Step 2: We solve for the equilibrium under full-information ($\psi = 1$) during the transition path.

Step 3: We apply the previous result to the balanced growth path to derive explicit solutions and comparative statics.

Step 4: We describe the link between prices and market tightness under a fully revealing separating equilibrium.

Step 5: We construct the unique, fully revealing separating equilibrium during the transition path recursively. We proceed in two substeps:

(a) First, we show that, under certain conditions, the full-information allocation can also be sustained when $\psi < 1$.

(b) Second, we prove the existence and uniqueness of a separating equilibrium in the case where the full-information optimum is not part of the possible strategies under asymmetric information.

Step 6: We apply the result from the previous step to a balanced growth path and conclude the proof.

Step 7: We show that there cannot be a pooling equilibrium if $a \to \phi q^P(\omega, a, \hat{a}, q) - \nu^b(\omega a)$ is monotonous in a during the transition path.

Step 8: We show that there is no pooling equilibrium on the balanced growth path by applying the previous step.

Step 1: D1 criterion and link between prices and average expected quality The definition of a Bayesian equilibrium does not impose any constraint on off-path beliefs, that is, on beliefs over events that have probability zero once a given strategy is chosen by sellers of a certain type. This can generate multiple equilibria that are not robust to small perturbations. For this reason, we impose more structure on beliefs by imposing the D1 criterion of Cho and Kreps (1987). This criterion is based on finding the set of types who are most likely to deviate relative to given on-path strategies. Then, it requires that operating firms have beliefs consistent with this set after observing the deviation. Finally, the D1 criterion allows to eliminate all allocations in which a seller's payoff from the deviation is not dominated under the worst possible consistent belief of buyers. Let us first define the relevant objects to our analysis.

For this, we drop the time subscripts as there is no ambiguity and fix an observed quality $\omega \in \Omega$ so that all definitions below are given conditional on ω. We also fix a given Bayesian equilibrium without further refinement.

For all $a \in \mathcal{A}$, let us denote $\mathcal{V}(a)$ the value of the seller of quality a in equilibrium, $q(a)$ its price, and $\hat{a}$ the quality it announces.

Let $q(\pi, \theta)$ be the price defined by the indifference condition of buyers conditional on tightness θ and buyers' belief function π. For all unobserved qualities $a_i \in \mathcal{A}$, let $\Delta_i(\tilde{a}, \theta)$ be the set of belief functions such that for all $\pi \in \Delta_i(\tilde{a}, \theta)$, deviating from its equilibrium strategy to the submarket $(\tilde{a}, q(\pi, \theta))$ is profitable for the seller of quality a_i.

Definition* 5. *Worst possible consistent beliefs

We define the buyers' worst possible consistent beliefs for a given equilibrium, an announced quality $\tilde{a}$, and a tightness θ as the quality a_i for $i \in [1, N_a]$ such that for all $a_j \in \mathcal{A}$, $\Delta_j \subset \Delta_i$. That is, the seller of quality a_i is better off under the largest set of beliefs in the sense of inclusion.

By definition of the D1 criterion, buyers will set their beliefs according to the worst possible consistent belief. That is, for a given announced quality $\hat{a}$ and market tightness θ, they will expect the unobserved quality that is better off under the largest set of beliefs. The previous definition formalizes this "largest" set of beliefs in terms of belief functions. The issue with the definition above is that it might not be possible to order the set of beliefs that make the seller of quality a_i and the seller of quality a_j better off since the two sets might contain an element that is absent from each other. This difficulty arises because set inclusion is not a complete order. In order to make sure that we can always find the "worst consistent belief" of a buyer, we need to make an assumption that will guarantee that the order is complete.

We make the following assumption, which guarantees that under asymmetric information, the price in a given submarket is increasing in the average expected quality expected by buyers :

***Assumption* 2.** *We assume that for all $(\theta, q, \tilde{a}) \in \mathbb{R}+^2 \times \mathcal{A}$, the function $a' \to \nu^b(a') - \psi q^P(a', \tilde{a}, q)$ is increasing in a'.*

We now characterize the set of beliefs consistent with the D1 criterion.

***Lemma* 1.** *Suppose that Assumption 2 holds. Given a Bayesian perfect equilibrium, consider any off-equilibrium submarket denoted by $(\hat{a}, \theta)$. Then, the worst possible consistent belief of buyers is* *well defined. It is the unobserved quality whose seller has a profitable deviation at $(\hat{a}, \theta)$ under the worst possible average expected quality.*

Proof. Let $a \in \mathcal{A}$ be a given unobserved quality, $\hat{a} \in \mathcal{A}$ a given announced quality, π be a belief function and $\theta \in \mathbb{R}+$ a given tightness. Using the indifference condition of buyers we have that

$$(1-\psi)q(\hat{a}, \theta, \pi) = \sum_{a' \in \mathcal{A}} \pi(a')[\nu^{\mathrm{b}}(a') - \psi q^P(a', \hat{a}, q(\hat{a}, \theta, \pi))] - \frac{\chi}{\mu(\theta)}.$$

Since for a fixed q, $\nu^{\mathrm{b}}(a') - \psi q^P(a', \hat{a}, q)$ is increasing in a' and $-\psi q^P(a', \hat{a}, q)$ is decreasing in q, we obtain by the implicit function theorem that the term between brackets is increasing in a'. This, in turn, implies that $q(\tilde{a}, \theta, \pi)$ is increasing in average quality $a_\pi \equiv \sum_{a \in \mathcal{A}} a' \pi(a')$, which is a sufficient statistic for the dependency on π.

Type a has a profitable deviation on $(\hat{a}, \theta)$ if and only if

$$\mathcal{V}(a) \leq p(\theta)\left[(1-\psi)q(\hat{a}, \theta, a_\pi) + \psi q^P(a, \hat{a}, q(\hat{a}, \theta, a_\pi)) - \nu^{\mathrm{s}}(a) + \delta\omega a\right] + \nu^{\mathrm{s}}(a) + \delta\omega a.$$

The right-hand side is then increasing in average quality a_π. Hence, $\Delta_k(\hat{a}, \theta)$ has the form $\{\pi : a \to [0,1] | \sum_{a' \in \mathcal{A}} \pi(a') = 1 \text{ and } a_\pi > x\}$ for some x. This, in turn, implies that the worst possible consistent belief on submarket $(\hat{a}, \theta)$ is the quality that has a profitable deviation under the lowest possible average quality expected by buyers. The form of Δ_i also implies that Δ_i and Δ_j can always be ordered by inclusion. ■

Step 2: Transitional dynamics in the full-information case We now characterize the equilibrium under full-information, namely the equilibrium when the signal is always informative of the unobserved quality ($\psi = 1$).

We fix the continuation value of sellers $\nu^{\mathrm{s}}_{t+1}(a)$ for all $a \in \mathcal{A}$ and our given observed quality ω, and describe the equilibrium in period t conditional on agents' continuation values. From Proposition 1, we have that the values of sellers and buyers are given by

$$\nu^{\mathrm{b}}_t(\omega, a) = (Z_t - \delta)\omega a + \Lambda_t\left((1-\varphi)\nu^{\mathrm{b}}_{t+1} + \varphi\nu^{\mathrm{s}}_{t+1}(\omega, a)\right)$$

and

$$\nu^{\mathrm{s}}_t = p(\theta^{FI}_t(\omega, a))\eta\nu^b_t(\omega, a) + (1 - p(\theta^{FI}_t(\omega, a)))(\Lambda_{t,t+1}\nu^{\mathrm{s}}_{t+1}(\omega, a) - \delta\omega a).$$

Under full-information, the first-order condition with respect to vacancies posted is given by

$$\mu_t(\theta_t(\omega, a, q_t))(\nu^b_t(\omega, a) - q^{FI}_t(\omega, a)) = w_t$$

, which relates the expected benefit from searching in a given submarket with the expected cost, and provides an indifference condition between sale prices and trading probabilities. Given this condition, the seller's maximization problem is then given by

$$\max_{\theta} p(\theta)\left(\nu_t^{\mathrm{b}}(\omega,a)-\frac{w_t\theta}{\gamma^t p(\theta)}\right)+(1-p(\theta))(\Lambda_{t,t+1}\nu_{t+1}^{\mathrm{s}}(\omega,a)-\delta\omega a),$$

which gives the first-order condition with respect to θ:

$$p'(\theta)(\nu_t^{\mathrm{b}}(\omega,a)-\Lambda_{t,t+1}\nu_{t+1}^{\mathrm{s}}(\omega,a)+\delta\omega a)=\frac{w_t}{\gamma^t}$$

We then replace the right-hand side using the indifference condition of buyers to obtain

$$(1-\eta)(\nu_t^{\mathrm{b}}(\omega,a)-\Lambda_{t,t+1}\nu_{t+1}^{\mathrm{s}}(\omega,a)+\delta\omega a)=\nu_t^{\mathrm{b}}(\omega,a)-q_t^{FI}(\omega,a),$$

from which we can solve for the equilibrium full-information price

$$q_t^{FI}(\omega,a)=\eta\nu_t^{\mathrm{b}}(\omega,a)+(1-\eta)(\Lambda_{t,t+1}\nu_{t+1}^{\mathrm{s}}(\omega,a)-\delta\omega a).$$

To find the associated optimal market tightness, we replace this price into the seller's first-order condition and solve for market tightness to obtain

$$\theta_t^{FI}(\omega,a)=\left(\frac{\bar{m}\gamma^t}{w_t}(1-\eta)(\nu_t^{\mathrm{b}}(\omega,a)-\Lambda_{t,t+1}\nu_{t+1}^{\mathrm{s}}(\omega,a)+\delta\omega a\right)^{1/\eta}.$$

Finall, we replace the expression for the optimal price and market tightness in the expression of the seller's value to obtain

$$\nu_t^{\mathrm{s}}(\omega,a)=p(\theta_t^{FI}(\omega,a))\eta\nu_t^{b}(\omega,a)+(1-p(\theta_t^{FI}(\omega,a))\eta)(\Lambda_t\nu_{t+1}^{S}(\omega,a)-\delta\omega a).$$

Step 3: Balanced growth path under full-information We can now use the result from Step 2 to obtain closed-form solutions for values and terms of trade on the balanced growth path equilibrium.

Let $\chi=\frac{w_t}{\gamma^t}$ denote the detrended wage in the balanced growth path. From our expression above we have

$$q^{FI}(\omega,a)=\eta\nu^{\mathrm{b}}(\omega,a)+(1-\eta)(\beta\nu^{\mathrm{s}}(\omega,a)-\delta\omega a)$$

and

$$\theta^{FI}(\omega,a)=\left(\frac{\bar{m}}{\chi}(1-\eta)(\nu^{\mathrm{b}}(\omega,a)-\beta\nu^{\mathrm{s}}(\omega,a)+\delta\omega a\right)^{1/\eta}.$$

From Proposition 1, the seller's and buyer's values in the balanced growth path under full-information are given by

$$\nu^{\mathrm{b}}(\omega,a) = (Z-\delta)\omega a + \beta\left[(1-\varphi)\nu^{\mathrm{b}}(\omega,a) + \varphi\nu^{\mathrm{s}}(\omega,a)\right],$$
$$\nu^{\mathrm{s}}(\omega,a) = q^{FI}(\omega,a)p(\theta^{FI}(\omega,a)) + \left(1-p(\theta^{FI}(\omega,a))\right)\left(\beta\nu^{\mathrm{s}}(\omega,a) - \delta\omega a\right).$$

Replacing these values into the optimal market tightness $\theta^{FI}(\omega,a)$, we obtain

$$p(\theta^{FI}(\omega,a)) = \bar{m}\left(\frac{Z\omega a\bar{m}(1-\eta)}{\chi\left(1-\beta(1-\varphi)(1-\eta p(\theta^{FI}(\omega,a)))\right)}\right)^{\frac{1-\eta}{\eta}},$$

We can derive the comparative static by differentiating with respect to ω:

$$\frac{dlogp(\theta^{FI}(\omega,a))}{dlog(\omega)} = \frac{1-\eta}{\eta} > 0.$$

Similarly, we replace the buyer's and seller's values into the optimal price $q^{FI}(\omega,a)$ to obtain

$$q^{FI}(\omega,a) = \frac{\omega a}{1-\beta}\left[\eta Z\frac{1-\beta(1-p(\theta^{FI}(\omega,a)))}{1-\beta(1-\varphi)(1-\eta p(\theta^{FI}(\omega,a)))} - \delta\right] \equiv \frac{\omega a}{1-\beta}F(p(\theta^{FI}(\omega,a))).$$

We can derive the comparative static by differentiating with respect to ω:

$$\frac{dq^{FI}(\omega,a)}{d\omega} = \frac{a}{1-\varphi}\left[F(p(\theta^{FI}(\omega,a))) + \omega F'(p(\theta^{FI}(\omega,a)))\frac{dp(\theta^{FI}(\omega,a))}{d\omega}\right],$$

where

$$F'(p(\theta^{FI}(\omega,a))) = \eta Z\beta\frac{1-(1-\varphi)[\eta+\beta(1-\eta)]}{(1-\beta(1-\varphi)(1-\eta p(\theta^{FI}(\omega,a))))^2} > 0.$$

Thus, when $q^{FI}(\omega,a) \geq 0$, we obtain $\frac{dq^{FI}(\omega,a)}{d\omega} > 0$. Since qualities ω and a have similar effects on optimal terms of trade, the same comparative statics apply to changes in a under full information.

Finally, we can express the value of sellers and buyers as

$$\nu^{\mathrm{s}}(\omega,a) = \frac{\omega a}{1-\beta}\left[\frac{\eta Zp(\theta^{FI}(\omega,a))}{1-\beta(1-\varphi)(1-\eta p(\theta^{FI}(\omega,a)))} - \delta\right]$$

and,

$$\nu^{\mathrm{b}}(\omega,a) = \frac{(Z-\delta)\omega a}{1-\beta(1-\varphi)} + \frac{\varphi\beta}{1-\beta(1-\varphi)}\nu^{\mathrm{s}}(\omega,a),$$

respectively, which proves our claims under full information.

Step 4: Prices and market tightness in a fully revealing separating equilibrium The following Lemma characterizes the equilibrium prices and market tightness in a fully revealing separating equilibrium.

Lemma* 2. *Equilibrium market tightness in a fully revealing separating equilibrium

In a fully revealing separating allocation sellers never misreport their true unobserved quality—i.e., $\hat{a}(\omega, a) = a \ \forall(\omega, a) \in \Omega \times A$. *Then, market tightness is given by*

$$\theta_t(\omega, \hat{a}(\omega, a), q(\omega, a)) = \mu^{-1}\left(\frac{w_t}{\nu^{\mathrm{b}}(\omega, a) - (1-\psi)q(\omega, a) - \psi q^P(\omega, a, \hat{a}, q)|q}\right) \tag{A.1}$$

Proof. In a fully revealing separating allocation, the vector $(\omega, \hat{a}, q)$ reveals the unobserved quality a by definition. Hence, using the indifference condition of buyers we obtain

$$\theta_t(\omega, \hat{a}(\omega, a), q(\omega, a)) = \mu^{-1}\left(\frac{w_t}{\mathbb{E}(\nu^{\mathrm{b}}(\omega, a) - (1-\psi)q(\omega, a) - \psi q^P(\omega, a, \hat{a}, q)|q, \hat{a})}\right)$$

Since the allocation is fully revealing, $\hat{a}(\omega, a) = a$ which implies $q^P(\omega, a, \hat{a}, q) = q$. We then obtain the result of Lemma 2. ∎

Step 5: Recursive construction of the unique fully revealing separating equilibrium Let us now consider the case with $\psi < 1$. We first define the notations that will be used within this step of the proof. In what follows, we fix $\omega \in \Omega$ and the time t. We omit all references to ω and to time t as there will be no ambiguity (all variables depend on t except for the continuation values taken at $t+1$). We note $\bar{v}(a) = \Lambda_{t,t+1}\nu^{\mathrm{s}}_{,t+1}(\omega, a) - \delta\omega a$ the continuation of a seller of unobserved quality a and observed quality ω. We also note $V(a) = p(\theta(\omega, q(\omega, a)))\left[\nu^{\mathrm{b}}(\omega, a) - \bar{v}(a)\right] - \chi\theta(\omega, q(\omega, a))$. Since the set $\mathcal{A}$ is ordered, so that $\mathcal{A} = \{a_i, i \in [1, ...N_a] \ |a_i > a_j \forall i > j\}$, we then note $\mathcal{A}_k = \{a \in \mathcal{A} | a \leq a_k\}$. Finally, we denote $q^B(a_i, a_j) = q^P(\omega, a_i, a_j) \forall(a_i, a_j) \in \mathcal{A}$ in the case where $q \geq q^P(\omega, a_i, a_j)$. That is, q^B is the post-inspection price conditional on having posted a price that is superior to $q^P(\omega, a_i, a_j)$. In general, we note $q^P(\omega, a_i, a_j, q) = min(q, q^B(a_i, a_j))$.

We will show the following Assertion by induction on $k \in [1, N_a]$.

***Assertion* 1. *Assertion at rank* $\mathbf{k} \in [\mathbf{1}, \mathbf{N_a}]$**

Let us construct the allocation $\Theta_k = \{\theta(a_1), \ldots, \theta(a_k)\}$ *on* $\mathcal{A}_k$ *such that the seller of quality* a_1 *implements the same strategy as under full information and* $\forall i \in [2, k]$, $\theta(a_i)$ *is constructed recursively using the procedure described for* a_k *below:*

(i) either for all $a_l < a_k$, $p(\theta_l)\left[\nu^{\mathrm{b}}(a_l) - \bar{v}(a_l)\right] - \chi\theta_l > p(\theta^{FI}(a_k))[(1-\psi)q^{FI}(a_k) + \psi q^B(a_l, a_k) - \bar{v}(a_l)] - (1-\psi)\chi\theta^{FI}(a_k)$ *and* $\theta_k = \theta^{FI}(a_k)$,

(ii) or the previous condition is not satisfied, and for all $l \leq k-1$ let

$$\bar{\theta}_l^k = \begin{cases} \text{the lowest solution } \theta\text{, if there is any, to} \\ p(\theta_l)\left[\nu^{\mathrm{b}}(a_l) - \bar{v}(a_l)\right] - \chi\theta_l = p(\theta^{FI}(a_k))[(1-\psi)q^{FI}(a_k) + \psi q^B(a_l, a_k) - \bar{v}(a_l)] - (1-\psi)\chi\theta \\ -\infty \text{ otherwise} \end{cases}$$

and let

$$\underline{\theta}_l^k = \begin{cases} \text{the largest solution } \theta\text{, if there is any, to} \\ p(\theta_l)\left[\nu^{\mathrm{b}}(a_l) - \bar{v}(a_l)\right] - \chi\theta_l = p(\theta^{FI}(a_k))[(1-\psi)q^{FI}(a_k) + \psi q^B(a_l, a_k) - \bar{v}(a_l)] - (1-\psi)\chi\theta \\ +\infty \text{ otherwise} \end{cases}$$

Let

$$\bar{\theta}_l^k \text{for } l = argmin\left\{\bar{\theta}_j^k,\ j \in [1, k-1]\right\}$$

and

$$\underline{\theta}_m^k \text{ for } m = argmax\left\{\underline{\theta}_j^k,\ j \in [1, k-1]\right\}$$

$\bar{\theta}_l^k$ and $\underline{\theta}_m^k$ are the two only possible values of θ_k in a separating equilibrium.

Suppose that $\bar{v}(a_k) \geq \bar{v}(a_l)$ for all $l \leq k$. Then there exists a unique fully revealing separating equilibrium. It is characterized by $\theta_k = \underline{\theta}_m^k$ and $\Theta_{k-1} \cup \{\theta_k\}$.

Initialization: $\mathcal{A}_1 = \{a_1\}$. We begin the construction by noting that since in the set $\mathcal{A}_1$ there is no lower type who needs to be disincentivized from mimicking for type a_1 and type a_1 does not want to mimic any higher type because there is none, we have that in any separating equilibrium in $\mathcal{A}_1$:

$$\begin{cases} \hat{a}(a_1) = a_1 \\ q(a_1) = q^{FI}(a_1) \end{cases}, \tag{A.2}$$

which proves the assertion for $k = 1$.

Recursion. Let us fix $k \in [2, N_a]$ and suppose that the assertion is true for $k-1$.

Step 5.(a): The full-information optimum can be sustained under asymmetric information. We first study the case in which the full-information price can be sustained for quality a_k. Given the sequence Θ_{k-1}, the full-information strategy of the seller of quality a_k can be part of a fully revealing separating equilibrium if and only if no seller of a lower quality wants to deviate from its current strategy to mimic him. Formally, the incentive compatibility constraint must be just binding or slack for every quality $a_i \leq a_k$ under the allocation Θ_{k-1} and $\theta(a_k) = \theta^{FI}(a_k)$. This yields

$$\begin{aligned} p(\theta^{FI}(a_i) \left[v^{\mathrm{b},FI}(a_i) - \Lambda\nu^{\mathrm{s},FI}(a_i) + \delta\omega a_i\right] - \chi\theta^{FI}(a_i) \geq \\ p(\theta^{FI}(a_k)) \left[(1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^P(a_i, a_k) - \Lambda\nu^{\mathrm{s},FI}(a_i) + \delta\omega a_i\right] - (1-\psi)\chi\theta^{FI}(a_k). \end{aligned} \tag{A.3}$$

The seller of quality a_k is then allowed to implement its full-information strategy which maximizes its unconstrained objective conditional on $\hat{a}(a_k) = a_k$.

Since a_k is the highest quality on $\mathcal{A}_k$, the seller would not be able to obtain a higher price by mimicking another quality. Then, since no seller has an incentive to deviate to mimic quality a_k, we can omit the incentive-compatibility constraint with respect to a_k in all other sellers' problem.

The last step is to discuss off-equilibrium beliefs conditional on being on a submarket where a_k is the announced quality. We can first rule out that quality a_k is expected by buyers. Indeed the seller of quality a_k is not better off deviating for any tightness θ conditional on being the expected quality or conditional on any lower quality being expected, as it is achieving its unconstrained optimum. If the seller of any other quality is better off deviating to a submarket $(\hat{a} = a_k, \theta)$ conditional on quality a_k being expected, then this seller would be better off under a larger set of beliefs than the seller of quality a_k. The D1 criterion would then impose that his quality is expected and not quality a_k, so that a deviation featuring the belief that a_k is deviating is ruled out by the D1 criterion. Hence, we are necessarily in a case where the expected quality is strictly lower than a_k.

Suppose now that a seller of some quality $a_i < a_k$ has a profitable deviation conditional on the announced quality being a_k and the expected quality being $a_j < a_k$. Then since $q^B(a_i, a_k) \leq q^B(a_i, a_j)$ the seller of quality a_i would have a profitable deviation if the set of qualities was $\mathcal{A}_{k-1}$ as well. Using the recursion at rank $k-1$, we know that this is not the case since Θ_{k-1} is an equilibrium on $\mathcal{A}_{k-1}$. As a consequence, no seller of any quality has a profitable deviation to both on-path or off-path submarkets where a_k is announced, and quality a_k is never expected by buyers on any off-equilibrium submarket.

Constructing the rest of the separating equilibrium on $\mathcal{A}_k$ is therefore equivalent to constructing the separating equilibrium on $\mathcal{A}_{k-1}$ as no seller of any other quality wants to mimic a_k nor to deviate and announce $\hat{a} = a_k$. Using the Assertion for $k-1$, we obtain a unique separating equilibrium where the allocation is $\Theta_k = \Theta_{k-1} \cup \{\theta^{FI}(a_k)\}$. This proves the first part of the assertion.

Step 5.(b): The full-information optimum cannot be sustained under asymmetric information. Let A_{k-1} denote the set of qualities that want to mimic sellers of quality a_k when they play their full-information market tightness. We now have that for all $j \in A_{k-1}$:

$$\begin{aligned} p(\theta_j)\left[\nu^{\mathrm{b}}(a_j) - \Lambda\nu^{\mathrm{s}}(a_j) + \delta\omega a_j\right] - \chi\theta_j < \\ p(\theta^{FI}(a_k))\left[(1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^P(a_j, a_k) - \Lambda\nu^{\mathrm{s}}(a_j) + \delta\omega a_j\right] - (1-\psi)\chi\theta^{FI}(a_k). \end{aligned} \tag{A.4}$$

Let $R_j^k(\theta) = p(\theta)\left[(1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^P(a_j, a_k, q(\theta, a_k)) - \bar{v}(a_j)\right] - (1-\psi)\chi\theta$. Then, $R_j^k(\theta) + \bar{v}(a_j)$ represents the revenue that the seller of quality a_j gets if it mimics the seller of quality a_k while the latter plays θ. We also denote $q(\theta, a_k) = \nu^{\mathrm{b}}(a_k) - \frac{\chi}{\mu(\theta)}$ the price given by the indifference condition of buyers as long as the allocation is separating for a_k.

Single-peaked shape of $R_j^k(\theta)$ We next analyze the properties of the function

$$R_j^k(\theta) = p(\theta)\left[(1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^P(a_j, a_k, q(\theta, a_k)) - \bar{v}(a_j)\right] - (1-\psi)\chi\theta$$

We have two cases to consider. Suppose first that $q(\theta, a_k) > q^B(a_j, a_k)$. Then $q^P(a_j, a_k, q(\theta, a_k)) = q^B(a_j, a_k)$ does not depend on θ. The second-order derivative with respect to θ is then

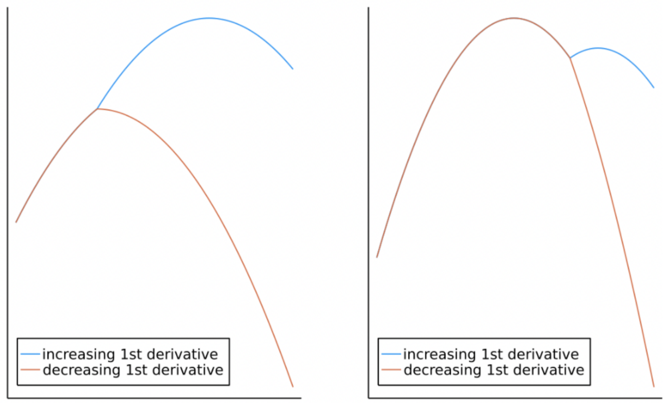
$$p''(\theta)\left[(1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^B(a_j, a_k) - \bar{v}(a_j)\right] < 0.$$

Hence, on this interval, the function is strictly concave. Now suppose that $q(\theta, a_k) \leq q^B(a_j, a_k)$. Then, $q^P(a_j, a_k, q(\theta, a_k)) = q(a_k, \theta)$ and the second-order derivative writes

$$p''(\theta)\left[\nu^{\mathrm{b}}(a_k) - \bar{v}(a_j)\right] < 0.$$

Hence, the revenue from mimicking is also strictly concave on that interval. In order to show that R_j^k is single-peaked, strictly increasing to the left of its maximum and strictly decreasing to the right of its maximum we need to look at the first order derivative at the junction. Indeed R_j^k is strictly concave on the invervals $q < q^B(a_j, a_k)$ and $q > q^B(a_j, a_k)$. Hence it is globally single-peaked if the first order derivative decreases at $q = q^B(a_j, a_k)$. A visual example is given in Figure 1 below.

FIGURE 1: Illustration of the junction of two concave functions



Note: This figure shows two cases : On the left panel the junction happens before the maximum of the first concave function. In this case no matter whether the first order derivative increases or decreases at the junction, the global function is single-peaked. On the right panel the junction happens after the maximum of the first concave function. We then need the first order derivative to decrease in order to obtain a single peak.

Let us now look at the first-order derivative. As long as $q(\theta, a_k) \geq q^B(a_j, a_k)$, we have $q^P(a_j, a_k, q(\theta, a_k)) = q^B(a_j, a_k)$. Hence, the first order derivative writes

$$p'(\theta)\left[(1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^B(a_j, a_k) - \bar{v}(a_j)\right] - (1-\psi)\chi.$$

We take the limit $q(\theta, a_k) \to q^B(a_j, a_k)$. At the limit, we solve for θ from the seller's indifference condition $q^B(a_j, a_k) = q(\theta, a_k) = \nu^b(a_k) - \frac{\chi}{\mu(\theta)}$. Replacing in the expression of the first-order derivative above, we find that the limit as $q(\theta, a_k) \to q^B(a_j, a_k)$ writes

$$p'(\theta)\left[\nu^b(a_k) - \bar{v}(a_j)\right] - (1-\psi)\chi - \psi\chi\theta$$

and for $q(\theta, a_k) \leq q^B(a_j, a_k)$, we have $q^P(a_j, a_k, q(\theta, a_k)) = q(\theta, a_k)$. Hence, the first-order derivative writes

$$p'(\theta)\left[\nu^{\mathrm{b}}(a_k) - \bar{v}(a_j)\right] - \chi.$$

As a consequence, the first-order derivative decreases strictly at the junction. Since q and θ vary in opposite directions (θ decreases as q gets larger), this implies that the revenue curve has one global maximum and is strictly increasing below that maximum and strictly decreasing above that maximum. Hence the revenue curve has a single-peaked shape.

Disincentivizing mimicking from lower types. Because the seller of quality a_k cannot play its full-information tightness, we have that for all $a_j \in A_{k-1}$:

$$R_j^k(0) = 0 < p(\theta_j)\left[\nu^{\mathrm{b}}(a_j) - \bar{v}(a_j)\right] - \chi\theta_j < R_j^k(\theta^{FI}(a_k)).$$

Using the inequality above and the fact that $R_j^k(\cdot)$ is continuous, for all $j \in A_{k-1}$ we can find $\tilde{\theta}_j < \theta^{FI}(a_k)$ such that

$$p(\theta_j)(\nu^{\mathrm{b}}(a_j) - \bar{v}(a_j)) - \chi\theta_j = R_j^k(\tilde{\theta}_j).$$

We then use the result that $R_j^k(\cdot)$ is single-peaked. This gives us that θ_j is on the strictly increasing part of $R_j^k(\cdot)$ because $R_j^k(\theta^{FI}(a_k)) > R_j^k(\tilde{\theta}_j)$. Hence, any value $\theta \leq \tilde{\theta}_j$ played by type a_k will disincentivize the seller of quality a_j from mimicking a_k. We note this value $\tilde{\theta}_j = \underline{\theta}_j$.

In addition, since $R_j^k(\cdot)$ is single-peaked and $\lim_{\theta \to +\infty} R_j^k(\theta) = -\infty$, there is at most one other value $\tilde{\theta}_j > \theta^{FI}(a_k)$ that satisfies the constraint. Since it is on the strictly decreasing part of $R_j^k(\cdot)$, any tightness higher than this one would disincentivize the seller of quality a_j from mimicking, which makes it a lower bound for a_k so that type a_j does not want to mimic. We note this value $\bar{\theta}_j$.

Then, let $\underline{\theta} = min_{i \leq k-1}(\underline{\theta}_i^k)$ and $\bar{\theta} = max_{i \leq k-1}(\bar{\theta}_i^k)$. We obtain that if type a_k plays $\theta \geq \bar{\theta}$ or $\theta < \underline{\theta}$, then no seller of a quality $a_j \in A_{k-1}$ wants to mimic the seller of quality a_k.

Other qualities $a_s \in \mathcal{A}_{k-1}$ A potential issue is that some other quality a_s might want to mimick a_k at $\bar{\theta}$ or $\underline{\theta}$ while not wanting to mimic at $\theta^{FI}(a_k)$. Suppose without loss that the seller of quality a_s wants to mimic at $\bar{\theta}$. We then have

$$R_s(0) = 0 < V(a_s) < R_s^k(\bar{\theta}).$$

Hence, we can construct $\tilde{\theta}_s$ such that

$$V(a_s) = R_s^k(\tilde{\theta}_s),$$

and because of the strict concavity of R_s, we have at most two solutions. Let us note $\underline{\theta}_s$ the lowest and $\bar{\theta}_s$ the highest of the two. We can then integrate these bounds into the construction of $\bar{\theta}$ and $\underline{\theta}$, which is exactly the same as previously. This, in turn, ensures that $a_s \in \mathcal{A}_{k-1}$ does not want to mimic at $\theta \geq \bar{\theta}$ nor at $\theta \leq \underline{\theta}$.
This reasoning can be iterated until we do not find any quality that wants to mimic at $\bar{\theta}$ or $\underline{\theta}$. Finally note that $\bar{\theta} > \theta^{FI}(a_k)$ and $\underline{\theta} < \theta^{FI}(a_k)$ as the bounds constructed in the previous step satisfied these inequalities and are updated if $\bar{\theta}_s > \bar{\theta}$ or $\underline{\theta}_s < \underline{\theta}$.

Any tightness $\theta \in (0, \underline{\theta}] \cup [\bar{\theta}, +\infty)$ is a possible value for a separating equilibrium. We just proved that for any $\theta \in (0, \underline{\theta}] \cup [\bar{\theta}, +\infty)$ played by the seller of quality a_k, all types in $\mathcal{A}_{k-1}$ do not want to mimic, which ensures that a separating equilibrium can be constructed using any of these values. Note that without a further refinement on off-equilibrium beliefs, we can always find a set of beliefs for buyers that can sustain any value below the upper bound found in the previous step as an equilibrium. We can, for example, suppose that the buyers believe that any off-path value is only picked by the lowest type and make any allocation a separating equilibrium. Hence, we need the D1 criterion in order to isolate the "most relevant" equilibrium.

The only values consistent with the D1 criterion are $\bar{\theta}$ or $\underline{\theta}$. Suppose that $\theta_k \neq \{\bar{\theta}_k, \underline{\theta}_k\}$. Let $a \in \mathcal{A}$, $a < a_k$. The fact that $\theta < \bar{\theta}$ or $\theta > \underline{\theta}$ implies that the constraint "quality a does not want to mimic type a_k" is not binding, i.e.,

$$p(\theta(a)) \left(\nu^{\mathrm{b}}(a) - \Lambda\nu^{\mathrm{s}}(a) + \delta\omega a\right) - \chi\theta(a) + \Lambda\nu^{\mathrm{s}}(a) - \delta\omega a >$$
$$p(\theta_k) \left((1-\psi)\nu^{\mathrm{b}}(a_k) + \psi q^P(a, a_k) - \Lambda\nu^{\mathrm{s}}(a) + \delta\omega a\right) - (1-\psi)\chi\theta_k + \Lambda\nu^{\mathrm{s}}(a) - \delta\omega a.$$

Let us start with the case $\theta_k < \underline{\theta}$. We now need to determine which seller is most likely to deviate and choose $(\hat{a} = a_k, \underline{\theta})$, so that we can set beliefs in accordance with the D1 criterion. We know that the seller of quality a_k is strictly better off if the price is $q(a_k, \underline{\theta})$, as $\theta_k < \underline{\theta}$ and values for tightness are on the strictly increasing part of the unconstrained revenue curve of the seller of quality a_k. At the same time, for any seller of a quality lower than a_k, we know by construction that if the price is $q(a_k, \underline{\theta})$, then they are not better off deviating. This implies that the seller of quality a_k is better off under a larger set of beliefs as he is better off for a larger set of prices in the sense of inclusion. The D1 criterion then implies that type a_k is the one expected on the whole interval $(0, \underline{\theta}]$. The symmetric reasoning implies that quality a_k is also expected on $[\bar{\theta}, +\infty)$.

We then invoke the fact that the unconstrained objective of sellers of quality a_k is strictly increasing on $(0, \underline{\theta}]$ because $\underline{\theta} < \theta^{FI}(a_k)$ and strictly decreasing on $[\bar{\theta}, +\infty)$ because $\bar{\theta} > \theta^{FI}(a_k)$. As a consequence, the seller of quality a_k has a profitable deviation for any $\theta_k < \underline{\theta}$ and $\theta_k > \bar{\theta}$, which leaves only $\bar{\theta}$ and $\underline{\theta}$ as possible equilibrium values.

Let us now show that if continuation values are increasing in unobserved quality, the seller of quality a_k prefers $\underline{\theta}$ to $\bar{\theta}$.

The seller of quality a_k prefers $\underline{\theta}$ to $\bar{\theta}$ if continuation values are increasing First it is trivial that the seller of quality a_k prefers $\underline{\theta}$ to $\bar{\theta}$ if $\bar{\theta} = +\infty$. We will therefore focus on cases where $\bar{\theta}$ is finite.

We now proceed in two steps. We first show that the seller of quality a_k has a higher value at $\underline{\theta}$ if the incentive-compatibility constraint is binding with the same quality for $\bar{\theta}$ and $\underline{\theta}$. We will then show that the same is true if the incentive-compatibility constraint binds with a different quality for $\bar{\theta}$ and $\underline{\theta}$.

(i) Case 1 : The same quality binds at $\bar{\theta}$ and $\underline{\theta}$

Let us start with the case where he same quality is binding with quality a_l on both sides. Let this quality be a_l. Let $\nu^S(\theta)$ be the value of the seller of quality a_k at θ.
We start with the sub-case where $q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) = q^P(a_l, a_k, q(a_k, \bar{\theta}_l^k))$, which is equivalent to $q(a_k, \bar{\theta}_l^k) \geq q^P(a_l, a_k, q(a_k, \bar{\theta}_l^k))$.

Using the incentive-compatibility constraint :

$$V(a_l) = (1-\psi)\nu^S(\underline{\theta}_l^k) - (1-\psi)\bar{\nu}(a_k) + p(\underline{\theta}_l^k)\left[\psi q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) + (1-\psi)\bar{\nu}(a_k) - \bar{\nu}(a_l)\right]$$

And the same relationship applies for $\bar{\theta}_l^k$. Substracting the expression above ot its counterpart at $\bar{\theta}_l^k$:

$$\begin{aligned}(1-\psi)\left[\nu^S(\underline{\theta}_l^k) - \nu^S(\bar{\theta}_l^k)\right] = p(\bar{\theta}_l^k)&\left[\psi q^P(a_l, a_k, q(a_k, \bar{\theta}_l^k)) + (1-\psi)\bar{\nu}(a_k) - \bar{\nu}(a_l)\right] \\ -p(\underline{\theta}_l^k)&\left[\psi q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) + (1-\psi)\bar{\nu}(a_k) - \bar{\nu}(a_l)\right]\end{aligned}$$

$$\begin{aligned}(1-\psi)\left[\nu^S(\underline{\theta}_l^k) - \nu^S(\bar{\theta}_l^k)\right] = \left(p(\bar{\theta}_l^k) - p(\underline{\theta}_l^k)\right)&\left[\psi q^P(a_l, a_k, q(a_k, \bar{\theta}_l^k)) + (1-\psi)\bar{\nu}(a_k) - \bar{\nu}(a_l)\right] \\ +\psi p(\underline{\theta}_l^k)&\left(q^P(a_l, a_k, q(a_k, \bar{\theta}_l^k)) - q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k))\right)\end{aligned}$$

The second term is zero given the sub-case we picked. We obtain :

$$(1-\psi)\left[\nu^S(\underline{\theta}_l^k) - \nu^S(\bar{\theta}_l^k)\right] = \left(p(\bar{\theta}_l^k) - p(\underline{\theta}_l^k)\right)\left[\psi q^P(a_l, a_k, q(a_k, \bar{\theta}_l^k)) + (1-\psi)\bar{\nu}(a_k) - \bar{\nu}(a_l)\right]$$

the right-hand side is positive, hence the left-hand side as well : $\underline{\theta}_l^k$ is preferred.
Let us now handle the sub-case $q^P(a_l, a_k, q(a_k, \bar{\theta}_l^k)) = q(a_k, \bar{\theta}_l^k)$. We have :

$$V(a_l) = \nu^S(\bar{\theta}_l^k) - \bar{\nu}(a_k) + p(\bar{\theta}_l^k)\left[\bar{\nu}(a_k) - \bar{\nu}(a_l)\right]$$

And :

$$V(a_l) = \nu^S(\underline{\theta}_l^k) - \bar{\nu}(a_k) + p(\underline{\theta}_l^k)\left[\psi(q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) - q(a_k, \underline{\theta}_l^k)) + \bar{\nu}(a_k) - \bar{\nu}(a_l)\right]$$

Substracting the two expressions :

$$\left(\nu^S(\underline{\theta}_l^k) - \nu^S(\bar{\theta}_l^k)\right) = p(\bar{\theta}_l^k)\left[\bar{\nu}(a_k) - \bar{\nu}(a_l)\right]$$
$$-p(\underline{\theta}_l^k)\left[\psi(q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) - q(a_k, \underline{\theta}_l^k)) + \bar{\nu}(a_k) - \bar{\nu}(a_l)\right]$$

$$\left(\nu^S(\underline{\theta}_l^k) - \nu^S(\bar{\theta}_l^k)\right) = \left(p(\bar{\theta}_l^k) - p(\underline{\theta}_l^k)\right)\left[\bar{\nu}(a_k) - \bar{\nu}(a_l)\right]$$
$$+p(\underline{\theta}_l^k)\psi\left[q(a_k, \underline{\theta}_l^k) - q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k))\right] \geq 0$$

Hence $\underline{\theta}$ is chosen in the case where the incentive-compatibility constraint binds with the same quality on both sides.

(ii) Case 2 : Different qualities bind at $\bar{\theta}$ and $\underline{\theta}$

Let us now take $l, m < k$ such that $\bar{\theta} = \bar{\theta}_m^k$ and $\underline{\theta} = \underline{\theta}_l^k$. Suppose that $m \neq l$.

By definition of the bounds, we then have :

$$\begin{cases} \bar{\theta}_l^k < \bar{\theta}_m^k \\ \underline{\theta}_l^k < \underline{\theta}_k^k \end{cases}$$

Since $\bar{\theta}_l^k$ is on the decreasing part of R_l^k and $\bar{\theta}_l^k < \bar{\theta}_m^k$:

$$R_l^k(\bar{\theta}_l^k) > R_l^k(\bar{\theta}_m^k)$$

Which by definition of $\bar{\theta}_l^k$ yields :

$$V(a_l) > R_l^k(\bar{\theta}_m^k)$$

We substract $V(a_m)$ on both sides to obtain :

$$V(a_l) - V(a_m) > p(\bar{\theta}_m^k)\left[\psi(q^P(a_l, a_k, q(a_k, \bar{\theta}_m^k)) - q^P(a_m, a_k, q(a_k, \bar{\theta}_m^k)) + \bar{\nu}(a_m) - \bar{\nu}(a_l))\right] \tag{A.5}$$

Let us now compare the value obtained by the seller of quality a_k at the two values $\bar{\theta}_m^k = \bar{\theta}$ and $\underline{\theta}_l^k = \underline{\theta}$. Let us denote $\nu^S(\theta)$ the value of the seller of quality a_k at θ.
We start with the sub-case $q^P(a_l, a_k, q(a_k, \bar{\theta}_m^k)) < q(a_k, \bar{\theta}_m^k)$.

The binding incentive-compatibility constraint with respect to quality a_l yields :

$$V(a_l) = (1-\psi)\nu^S(\underline{\theta}_l^k) - (1-\psi)\bar{\nu}(a_k) + p(\underline{\theta}_l^k)\left[\psi q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) + (1-\psi)\bar{\nu}(a_k) - \bar{\nu}(a_l)\right]$$

We subtract the above expression to its counterpart at $\bar{\theta}_m^k$ and obtain :

$$\begin{aligned} (1-\psi)\left[\nu^S(\underline{\theta}_l^k) - \nu^S(\bar{\theta}_m^k)\right] &= V(a_l) - V(a_m) \\ +p(\bar{\theta}_m^k)\left[\psi q^P(a_m, a_k, q(a_k, \bar{\theta}_m^k)) + (1-\psi)\bar{\nu}(a_k) - \bar{\nu}(a_m)\right] \\ -p(\underline{\theta}_l^k)\left[\psi q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) + (1-\psi)\bar{\nu}(a_k) - \bar{\nu}(a_l)\right] \end{aligned}$$

Using inequality A.5 above, we obtain :

$$\begin{aligned} (1-\psi)\left[\nu^S(\underline{\theta}_l^k) - \nu^S(\bar{\theta}_m^k)\right] > \\ p(\bar{\theta}_m^k)\left[\psi q^P(a_l, a_k, q(a_k, \bar{\theta}_m^k)) + (1-\psi)\bar{\nu}(a_k) - \bar{\nu}(a_l)\right] \\ -p(\underline{\theta}_l^k)\left[\psi q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) + (1-\psi)\bar{\nu}(a_k) - \bar{\nu}(a_l)\right] \end{aligned}$$

$$\begin{aligned} (1-\psi)\left[\nu^S(\underline{\theta}_l^k) - \nu^S(\bar{\theta}_m^k)\right] > (p(\bar{\theta}_m^k) - p(\underline{\theta}_l^k))\left[\psi q^P(a_k, a_l, q(a_k, \underline{\theta}_l^k)) + (1-\psi)\bar{\nu}(a_k) - \bar{\nu}(a_l)\right] \\ +\psi p(\bar{\theta}_m^k)\left[q^P(a_l, a_k, q(a_k, \bar{\theta}_m^k)) - q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k))\right] \end{aligned}$$

In this sub-case the second term on the right-hand side is zero. The first term on the right-hand side is positive since $q^P(a_l, a_k, q(a_k, \underline{\theta}_l^k)) \geq \bar{\nu}(a_l)$. Hence $\nu^S(\underline{\theta}_l^k) - \nu^S(\bar{\theta}_m^k) \geq 0$: $\underline{\theta}$ is chosen.

We now address the second sub-case, namely $q^P(a_l, a_k, q(a_k, \bar{\theta}^k_m)) = q(a_k, \bar{\theta}^k_m)$.

Using the incentive-compatibility constraint :

$$V(a_m) = \nu^S(\bar{\theta}^k_m) - \bar{\nu}(a_k) + p(\bar{\theta}^k_m)\left[\bar{\nu}(a_k) - \bar{\nu}(a_m)\right]$$

$$V(a_l) = \nu^S(\underline{\theta}^k_l) - \bar{\nu}(a_k) + p(\underline{\theta}^k_l)\left[\psi(q^P(a_l, a_k, q(a_k, \underline{\theta}^k_l)) - q(a_k, \underline{\theta}^k_l)) + \bar{\nu}(a_k) - \bar{\nu}(a_l)\right]$$

Hence we obtain :

$$\begin{aligned}\left[\nu^S(\underline{\theta}^k_l) - \nu^S(\bar{\theta}^k_m)\right] &= V(a_l) - V(a_m) + p(\bar{\theta}^k_m)\left[\bar{\nu}(a_k) - \bar{\nu}(a_m)\right]\\ &-p(\underline{\theta}^k_l)\left[\psi(q^P(a_l, a_k, q(a_k, \underline{\theta}^k_l)) - q(a_k, \underline{\theta}^k_l)) + \bar{\nu}(a_k) - \bar{\nu}(a_l)\right]\end{aligned}$$

We use inequality A.5 :

$$\begin{aligned}\left[\nu^S(\underline{\theta}^k_l) - \nu^S(\bar{\theta}^k_m)\right] \geq p(\bar{\theta}^k_m)\left[\psi(q^P(a_l, a_k, q(a_k, \bar{\theta}^k_m)) - q^P(a_m, a_k, q(a_k, \bar{\theta}^k_m)) + \bar{\nu}(a_m) - \bar{\nu}(a_l))\right]\\ +p(\bar{\theta}^k_m)\left[\bar{\nu}(a_k) - \bar{\nu}(a_m)\right]\\ -p(\underline{\theta}^k_l)\left[\psi(q^P(a_l, a_k, q(a_k, \underline{\theta}^k_l)) - q(a_k, \underline{\theta}^k_l)) + \bar{\nu}(a_k) - \bar{\nu}(a_l)\right]\end{aligned}$$

$$\begin{aligned}\left[\nu^S(\underline{\theta}^k_l) - \nu^S(\bar{\theta}^k_m)\right] &\geq \left(p(\bar{\theta}^k_m) - p(\underline{\theta}^k_l)\right)(\bar{\nu}(a_k) - \bar{\nu}(a_l))\\ p(\bar{\theta}^k_m)&\left[\psi(q^P(a_l, a_k, q(a_k, \bar{\theta}^k_m)) - q^P(a_m, a_k, q(a_k, \bar{\theta}^k_m)))\right]\\ &+\psi p(\underline{\theta}^k_l)\left[q(a_k, \underline{\theta}^k_l) - q^P(a_l, a_k, q(a_k, \underline{\theta}^k_l))\right]\end{aligned}$$

We use our sub-case expression $q^P(a_l, a_k, q(a_k, \bar{\theta}^k_m)) = q(a_k, \bar{\theta}^k_m)$:

$$\begin{aligned}\left[\nu^S(\underline{\theta}^k_l) - \nu^S(\bar{\theta}^k_m)\right] &\geq \left(p(\bar{\theta}^k_m) - p(\underline{\theta}^k_l)\right)(\bar{\nu}(a_k) - \bar{\nu}(a_l))\\ &\psi p(\bar{\theta}^k_m)\left[(q(a_k, \bar{\theta}^k_m) - q^P(a_k, a_m, q(a_k, \bar{\theta}^k_m)))\right]\\ &+\psi p(\underline{\theta}^k_l)\left[q(a_k, \underline{\theta}^k_l) - q^P(a_l, a_k, q(a_k, \underline{\theta}^k_l))\right]\end{aligned}$$

All the terms on the right-hand side are positive, hence $\nu^S(\underline{\theta}^k_l) - \nu^S(\bar{\theta}^k_m) \geq 0$. $\underline{\theta}$ is chosen.

So far, we have proved that if continuation values are increasing, the only possible value for a separating equilibrium is $\underline{\theta}$. We also showed that under beliefs set in accordance with the D1 criterion, the seller of quality a_k is expected but has no profitable deviation on $(0, \underline{\theta}] \cup [\hat{\theta}, +\infty)$. This guarantees that no seller has an incentive to deviate to these sub-markets.

We now need to set beliefs on the whole interval $]\underline{\theta}, \bar{\theta}[$ in order to assess whether there will be a positive deviation for a seller of quality a_k or of any lower quality.

Quality a_k is not expected on $(\underline{\theta}, \bar{\theta})$ if continuation values are increasing To obtain the existence of a fully revealing separating equilibrium, we must now make sure that no seller has a profitable deviation at any tightness in the interval $]\underline{\theta}, \bar{\theta}[$. Let us first set beliefs on this interval.

Suppose that $\theta_k = \underline{\theta}$. By construction, we know that the seller of quality a_k and at least one other seller of quality $\underline{a} < a_k$ will be better off deviating to θ' such that $\bar{\theta} > \theta' > \underline{\theta}$ if expected quality at θ' is a_k. Let us denote by $\nu^{\mathrm{b}}(\hat{a})$ the expected value of the capital good bought on this submarket, and $\hat{q}^P$ the expected price paid in case of successful inspection. Using the indifference condition of buyers, the price that achieves this tightness is then q' such that $(1-\psi)q' + \psi \hat{q}^P = \nu^{\mathrm{b}}(\hat{a}) - \frac{\chi}{\mu(\theta')}$.

The revenue of the seller of quality a_k is

$$p(\theta') \left[q' - \bar{v}(a_k)\right] + \bar{v}(a_k).$$

Hence, the difference between the revenue of the seller of quality a_k conditional on these beliefs and the one at $\bar{\theta}$ is

$$\Delta_k = p(\theta')[q' - \bar{v}(a_k)] - p(\bar{\theta}) \left[q(a_k) - \bar{v}(a_k)\right].$$

Similarly, for the seller of quality $\underline{a}$, the gain from deviating conditional on these beliefs is

$$\underline{\Delta} = p(\theta')[(1-\psi)q' + \psi q^P(\underline{a}, a_k, q') - \bar{v}(\underline{a})] - p(\theta(\underline{a})) \left[\nu^{\mathrm{b}}(\underline{a}) - \bar{v}(\underline{a})\right] + \chi\theta(\underline{a}).$$

Let us now use the incentive-compatibility constraint, which yields

$$p(\bar{\theta})[(1-\psi)q(a_k) + \psi q^P(\underline{a}, a_k, q(a_k)) - \bar{v}(\underline{a})] - p(\theta(\underline{a})) \left[q(a) - \bar{v}(\underline{a})\right] = 0.$$

This yields

$$\underline{\Delta} = \epsilon + p(\theta')[(1-\psi)q' + \psi q^P(\underline{a}, a_k, q') - \bar{v}(\underline{a})] - p(\bar{\theta})[(1-\psi)q(a_k) + \psi q^P(\underline{a}, a_k, q(a_k)) - \bar{v}(\underline{a})]$$

We know that the breakeven price of the seller of quality a_k at θ', denoted by $\tilde{q}$, is such that

$$p(\theta')\tilde{q} = (p(\theta') - p(\bar{\theta}))\bar{v}(a_k) + p(\bar{\theta})q(a_k, \bar{\theta})$$

At this price, we obtain

$$\underline{\Delta} = \epsilon + (p(\theta') - p(\bar{\theta}))[(1-\psi)\bar{v}(a_k) - \bar{v}(\underline{a})] + p(\theta')\psi q^P(\underline{a}, a_k, q') - p(\bar{\theta})\psi q^P(\underline{a}, a_k, q(a_k, \bar{\theta}))$$

Suppose that $\tilde{q} \leq q^B(\underline{a}, a_k)$ so that $q^P(\underline{a}, a_k, q') = q'$. Then, we obtain

$$\underline{\Delta} = (p(\theta') - p(\bar{\theta}))[\bar{v}(a_k) - \bar{v}(\underline{a})] + p(\bar{\theta})\psi(q(a_k) - q^P(\underline{a}, a_k, q(a_k, \bar{\theta}))).$$

Hence, a sufficient condition for $\underline{\Delta} \geq 0$ is $\bar{v}(a_k) - \bar{v}(\underline{a}) \geq 0$. When we reach $\bar{\theta}$, the incentive compatibility constraint will be exactly binding.

Otherwise, we obtain

$$\underline{\Delta} = (p(\theta') - p(\bar{\theta}))[\psi q^B(\underline{a}, a_k)) + (1-\psi)\bar{v}(a_k) - \bar{v}(\underline{a})].$$

Hence, a sufficient condition for $\underline{\Delta} \geq 0$ is $\bar{v}(a_k) - \bar{v}(\underline{a}) \geq 0$ as $q^B(\underline{a}, a_k)) \geq \bar{v}(\underline{a})$ by assumption.

Hence, for any $\theta' \in (\underline{\theta}, \bar{\theta})$, at the break-even price of sellers of quality a_k, sellers of quality $\underline{a}$ have a strictly profitable deviation. This implies that sellers of quality $\underline{a}$ are better off under the worst possible consistent beliefs, which in turn ensures that they are the expected quality on this interval according to the D1 criterion. We will now focus on qualities strictly lower than a_k and show that no seller of such a quality has profitable deviation on this interval.

Profitable deviations on $(\underline{\theta}, \bar{\theta})$. Suppose that quality $a_i < a_k$ is expected on some interval $[\underline{\theta}^k_m, \underline{\theta}^k_l]$ for $m, l < k$. Then, we have two cases:

(i) If the seller of quality a_i implements its full information strategy, it will not have a strictly profitable deviation.

(ii) Otherwise, it has a binding incentive-compatibility constraint with some quality a_j.

Let us address the latter case. We are only concerned about the values $\theta' \in [\bar{\theta}^i_j, \underline{\theta}^i_j]$ as otherwise the deviation is not profitable for the seller of quality a_i conditional on being expected. Let us set beliefs on this interval. By construction, for $\theta' \in [\bar{\theta}^i_j, \underline{\theta}^i_j]$, the sellers of quality a_i and a_j are both better off for $q(a_k, \theta')$. Given a belief function expressed as before and a resulting price q', the revenue from the deviation for the seller of quality a_i is

$$\Delta_i = p(\theta') \left[(1-\psi)q' + \psi q^P(a_i, a_k, q') - \bar{v}(a_i)\right] - V(a_i).$$

The revenue from the deviation for the seller of quality a_j is

$$\Delta_j = p(\theta') \left[(1-\psi)q' + \psi q^P(a_j, a_k, q') - \bar{v}(a_j)\right] - V(a_j).$$

Hence, we have

$$\Delta_j - \Delta_i = V(a_i) - V(a_j) + p(\theta') \times \left[\psi(q^P(a_j, a_k, q') - q^P(a_i, a_k, q')) + \bar{v}(a_i) - \bar{v}(a_j)\right].$$

Let us use the incentive-compatibility constraint

$$V(a_j) - V(a_i) = p(\theta_i) \left[\psi(q^B(a_j, a_i) - q(a_i)) + \bar{v}(a_i) - \bar{v}(a_j)\right].$$

Replacing this in the previous equation, we obtain

$$\begin{aligned}\Delta_j - \Delta_i = p(\theta') \times \left[\psi(q^P(a_j, a_k, q') - q^P(a_i, a_k, q')) + \bar{v}(a_i) - \bar{v}(a_j)\right] \\ -p(\theta_i)[\psi(q^B(a_j, a_i) - q(a_i)) + \bar{v}(a_i) - \bar{v}(a_j)].\end{aligned}$$

First, if $q' \leq q^B(a_j, a_k)$, then the second factor of the first term is larger than second factor of the second term. It will also be positive and, since $\theta' \geq \theta_i$, we will obtain that $\Delta_j \geq \Delta_i$.

The break-even price for the seller of quality a_i is such that

$$p(\theta')(1-\psi)q' = p(\theta_i)q(a_i) + (p(\theta') - p(\theta_i))\bar{v}(a_i) - \psi p(\theta')q^P(a_i, a_k, q').$$

We assume as before that $q' \geq q^B(a_j, a_k)$, as otherwise quality a_i would not be expected. At that price, we have

$$\Delta_j = p(\theta') \left[(1-\psi)q' + \psi q^P(a_j, a_k, q') - \bar{v}(a_j)\right] - V(a_j).$$

Again, the incentive-compatibility constraint gives

$$p(\theta_i)[(1-\psi)q(a_i) + q^B(a_j, a_i) - \bar{v}(a_j)] = V(a_j),$$

Which yields

$$\Delta_j = p(\theta') \left[(1-\psi)q' + \psi q^P(a_j, a_k, q') - \bar{v}(a_j)\right] - p(\theta_i) \left[(1-\psi)q(a_i) + q^B(a_j, a_i) - \bar{v}(a_j)\right].$$

Using the break-even price of sellers, this condition becomes

$$\Delta_j = (p(\theta') - p(\theta_i))(\bar{v}(a_i) - \bar{v}(a_j)) + \psi p(\theta_i)[q(a_i) - q^B(a_j, a_i)] - \psi p(\theta')[q^P(a_i, a_k, q') - q^B(a_j, a_k)]$$

This is an affine function of $p(\theta')$. Hence, we just need to check that it is positive for $\theta' = \theta_i$ and $\theta' = \bar{\theta}_i^j$. It is trivially positive at θ_i.

At $\bar{\theta}_j^i$, we know that the seller of quality a_j exactly breaks even if the expected quality is a_i. We will now use the recursion and the fact that a_i is better off at $\underline{\theta}_j^i$ than at $\bar{\theta}_j^i$. Hence, the seller of quality a_i at most breaks even, yielding a break-even price weakly higher than that of the seller of quality a_j. As a conclusion, if the seller of quality a_i is expected at θ', then necessarily he does not have a strictly profitable deviation as the only interval that allows for profitable deviations has the type with whom it binds be expected rather than him.

The last step before concluding the existence proof is to show that adding quality a_k does not change beliefs on submarkets where the signal is lower than a_k.

Adding qualities greater than a_k does not change beliefs for submarkets where the signal is $\hat{a} = a_k$. To show this, it is enough to notice that our previous reasonings hold even if we add a quality a_m higher than a_k. This is due to Lemma 1, which states that beliefs are set depending on the lowest average quality under which sellers have a profitable deviation. Adding a higher quality will not increase this statistic for any seller. If the seller of quality a_k plays a tightness in $[0, \underline{\theta}) \cup (\bar{\theta}, \infty)]$, he will still have a profitable deviation if quality a_m is expected instead of a_k.

Finally, the revenue from a deviation on $[\underline{\theta}, \bar{\theta}]$ will always be weakly lower for the seller of quality a_m than for the seller of quality a_k as they obtain the same price. This implies that the break-even price of sellers of quality lower than a_k will still be expected on this interval. Hence, beliefs will not be altered on this interval, nor the result that there is no strictly profitable deviation. Therefore, adding higher types preserves the equilibrium on submarkets with a strictly lower signal.

A non-revealing separating equilibrium is always weakly dominated by the fully revealing separating equilibrium. We can make our reasoning above for any signal $\hat{a} < a_k$ and similarly construct an equilibrium. As q^P is weakly decreasing in the signal, the revenue from mimicking for all lower types will be larger than for a signal a_k. This, in turn, implies that the bounds $\underline{\theta}_j^k$, conditional on signal $\hat{a} < a_k$, will be lower than for signal $\hat{a} = a_k$, and the bounds $\bar{\theta}_j^k$ will be larger, while the price obtained by the seller of quality a_k stays equal to the posted price (there is no penalty for announcing a quality lower than his own). This implies that the seller of quality a_k will have to pick a tightness further away from his full-information tightness than he would had he signaled quality a_k, yielding a lower payoff. This ensures that non-revealing separating equilibria are at least weakly dominated by the fully revealing equilibrium. Since we allow for q^P to be constant for all announcements, we cannot rule them out. If q^P were strictly increasing in the signal, then the fully revealing equilibrium would be the only one among the set of all separating equilibria.

Conclusion of the recursion. We proved that there exists a unique fully revealing equilibrium on $\Theta_{k-1} \cup \{\theta_k\}$ on $\mathcal{A}_k$ satisfying the D1 criterion, and that the construction is done using the procedure described in the assertion. Hence, the assertion is true at rank k, which concludes the induction.

Step 6: Balanced growth path under asymmetric information. The proof of Proposition 2 is then a simple application of the recursion above on $\mathcal{A} = \mathcal{A}_{N_a}$.

Step 7: There is no pooling equilibrium in transitional dynamics if $a \to \phi q^P(\omega, a, \hat{a}, q) - \nu^{\mathrm{b}}(\omega a)$ is monotonous in a. Assume that for all $\omega, \hat{a}, q \in \mathcal{A}^2 \times \mathbb{R}+$, $\psi q^P(\omega, a, \hat{a}, q) - \nu_{t+1}^{\mathrm{s}}(\omega, a) + \delta\omega a$ is monotonous in a. Suppose that there exists an equilibrium such that the subset of types $A \subset \mathcal{A}$ are pooled together in submarket $(\hat{a}, q)$ with some strictly positive probability. Let $\bar{a} = max(A)$. We suppose that they have the same observed quality ω and omit it for convenience.

We next show that under the D1 criterion, there exists a strictly profitable deviation for type $\bar{a}$, ruling out any equilibrium where pooling happens with strictly positive probability. Let θ be the tightness on submarket $(\hat{a}, q)$ where the pooling happens. We can assume without loss of generality that buyers visit this submarket. Otherwise, any type can deviate to any sub-market at which buyers would purchase for a strictly positive price, which would be a strictly profitable deviation. We then proceed by setting beliefs on submarkets where $\bar{a}$ is the signaled quality and θ' the market tightness using the D1 criterion.

Suppose that the seller of quality $\bar{a}$ is setting a different price while still announcing quality $\hat{a}$. It is better off for any price weakly greater than $\tilde{q}$ defined as

$$p(\theta')((1-\psi)\tilde{q} + \psi q^P(\bar{a}, \hat{a}, \tilde{q})) + (1-p(\theta'))\bar{v}(\bar{a}) = p(\theta)((1-\psi)q + \psi q^P(\bar{a}, \hat{a}, q)) + (1-p(\theta))\bar{v}(\bar{a}).$$

The seller of quality $a \in A$ is indifferent to any price weakly greater than $\underline{q}$ defined as

$$p(\theta')((1-\psi)\underline{q} + \psi q^P(a,\hat{a},\underline{q})) + (1-p(\theta'))\bar{v}(a) = p(\theta)((1-\psi)q + \psi q^P(a,\hat{a},q)) + (1-p(\theta))\bar{v}(a).$$

Suppose that $\theta' > \theta$ but close to θ. Then, given the above expression, we must have $\tilde{q}$ being slightly higher than q. If $\theta' < \theta$, then we must have $\tilde{q}$ slightly lower than q. Let us now set beliefs by fixing some price $\tilde{q}$. Let Δ be the benefit from deviating for the seller of quality a at tightness θ' and price $\tilde{q}$. Define $\bar{\Delta}$ similarly for quality $\bar{a}$. Then, we have

$$\bar{\Delta}-\Delta = (p(\theta)-p(\theta'))\left[\bar{v}(\bar{a}) - \bar{v}(a)\right] + p(\theta)\psi(q^P(a,\hat{a},q) - q^P(\bar{a},\hat{a},q)) - p(\theta')\psi(q^P(a,\hat{a},\tilde{q}) - q^P(\bar{a},\hat{a},\tilde{q})).$$

Let us take $\theta' < \theta$. Then, both $\tilde{q}$ must be larger than q. We obtain $q^P(a,\hat{a},q) - q^P(\bar{a},\hat{a},q) \leq q^P(a,\bar{a},\tilde{q}) - q^P(\bar{a},\hat{a},\tilde{q})$, which implies

$$\bar{\Delta} - \Delta \geq (p(\theta) - p(\theta'))\left[\psi(q^P(a,\hat{a},q) - q^P(\bar{a},\hat{a},q)) + \bar{v}(\bar{a}) - \bar{v}(a)\right].$$

We now use the assumption that $\psi q^P(a,\hat{a},q) - \bar{v}(a)$ is decreasing in a. This implies that the second term on the right-hand side is weakly higher than zero, as is the first term on the right-hand side. Hence, for all $a \in \mathcal{A}$ and $\theta' < \theta$, the seller of quality $\bar{a}$ always has a larger benefit from deviating than the seller of inferior quality, no matter what the price is. This ensures that the seller of quality $\bar{a}$ will be better off under the worst consistent beliefs. The D1 criterion then implies that quality $\bar{a}$ is expected at $\theta' < \theta$. We can now construct a positive deviation for the seller of quality $\bar{a}$. Using the indifference condition of buyers, we have that $(1-\psi)q = \mathbb{E}\left(\nu^{\mathrm{b}}(\bar{a}) - \psi q^P(a,\hat{a},q)|\hat{a},q\right) - \frac{\chi}{\mu(\theta)}$. Using Assumption 2, $\mathbb{E}\left(\nu^b(a) - \psi q^P(a,\hat{a},q|\hat{a},q)\right) = \nu^b(\bar{a}) - \psi q^P(\bar{a},\hat{a},q) - \epsilon$ for some $\epsilon > 0$. The revenue from deviating to tightness θ' for the seller of quality $\bar{a}$ is then

$$\bar{\Delta} = p(\theta')[\nu^b(\bar{a}) - \bar{v}(\bar{a})] - p(\theta)[\mathbb{E}\left(\nu^{\mathrm{b}}(a) - \psi q^P(a,\hat{a},q)|\hat{a},q\right) + \psi q^P(\bar{a},\hat{a},q) - \bar{v}(\bar{a})] + \chi(\theta-\theta'),$$

or

$$\bar{\Delta} = (p(\theta') - p(\theta))[\nu^{\mathrm{b}}(\bar{a}) - \bar{v}(\bar{a})] - \chi(\theta-\theta') + p(\theta)\epsilon.$$

Hence, we can find θ' sufficiently close to θ such that the deviation yields strictly positive profit, which in turn implies a strictly positive deviation for the seller of quality $\bar{a}$. Therefore, the equilibrium cannot be sustained. If the function $\psi q^P(a,\hat{a},q) - \bar{v}(a)$ is instead increasing in a, we make the exact symmetric reasoning with $\theta' > \theta$.

Step 8 : There is no pooling equilibrium on a balanced growth path Let us now apply the result from Step 7 to a balanced growth path equilibrium. Suppose that some unobserved qualities are pooled together at some announced quality $\hat{a}$ and price q. Let a be one of them. Let θ be the associated tightness.

The value of the seller of type a on the balanced growth path is :

$$\nu^{\mathrm{S}}(a) = p(\theta)((1-\psi)q + \psi q^{P}(a,\hat{a},q)) + (1-p(\theta))(\beta\nu^{\mathrm{S}}(a) - \delta\omega a)$$

$$\bar{\nu}^{\mathrm{S}}(a)(1-\beta(1-p(\theta))) = -\delta\omega a + p(\theta)((1-\psi)q + \psi q^{P}(a,\hat{a},q))$$

We can re-organize the previous expression to make the continuation value of the seller appear :

$$(\bar{\nu}(a) - \psi q^{P}(a,\hat{a},q))(1-\beta(1-p(\theta))) = -\delta\omega a + (1-\psi)p(\theta)q + \psi q^{P}(a,\hat{a})(p(\theta) - (1-\beta(1-p(\theta)))$$

Which allows us to express $\bar{\nu}(a) - \psi q^{P}(a,\hat{a},q)$:

$$\bar{\nu}(a) - \psi q^{P}(a,\hat{a},q) = \frac{1}{(1-\beta(1-p(\theta)))}\left[-\delta\omega a + (1-\psi)p(\theta)q - \psi q^{P}(a,\hat{a},q)(1+\beta)(1-p(\theta))\right]$$

Hence $a \to \bar{\nu}(a) - \psi q^{P}(a,\hat{a},q)$ is monotonous in a for all the unobserved qualities $a \in A$ that are pooled in submarket $(\hat{a},q)$. We can then apply the previous step, which implies that a pooling equilibrium is ruled out in this case and leads to a contradiction. Hence there is no pooling equilibrium on a balanced growth path.

■

A.1.1 Proof of proposition 4

In the case with $\mathcal{A} = \{a_1, a_2\}$ with $a_1 < a_2$, the seller of quality a_1 chooses the full-information price and market tightness. The strategy of the seller of quality a_2 is then determined by the binding incentive-compatibility constraint between him and sellers of quality a_1, which is given by

$$p(\theta^{FI}(a_1))(q^{FI}(a_1) - \beta\nu^{\mathrm{s}}(a_1) + \delta\omega a_1) = p(\theta(a_2))[(1-\psi)q(a_2) + \psi q^{P}(a_2, a_1, q(a_2)) - \beta\nu^{\mathrm{s}}(a_1) + \delta\omega a_1].$$

There exist a threshold ψ^{*} that triggers the non-full-information solution. As we showed before, a "small" degree of asymmetry of information implies that the constraint above is not binding.

Instead, the constraint becomes binding for a threshold value of ψ^* defined by

$$p(\theta^{FI}(a_1))(q^{FI}(a_1)-\beta\nu^{\mathrm{s}}(a_1)+\delta\omega a_1) = p(\theta^{FI}(a_2))[(1-\psi^*)q^{FI}(a_2)+\psi^* q^P(a_1,a_2,q^{FI}(a_2))-\beta\nu^{\mathrm{s}}(a_1)+\delta\omega a_1].$$

Since $q^P(a_2,a_1,q^{FI}(a_2)) < q^{FI}(a_1)$ by assumption, this condition will be met for some $\psi^* \in [0,1]$. The above equation can be re-written as

$$\begin{aligned}&\psi^* p(\theta^{FI}(a_2))(q^{FI}(a_2) - q^P(a_1,a_2,q^{FI}(a_2))) + \delta\omega(a_1-a_2) = \\ &\left(\nu^{S,FI}(a_2) - \nu^{S,FI}(a_1)\right)(1-\beta(1-p(\theta^{FI}(a_2)))),\end{aligned}$$

which in turn yields

$$\psi^* = \frac{\left(\nu^{S,FI}(a_2) - \nu^{S,FI}(a_1)\right)\left[1-\beta(1-p(\theta^{FI}(a_2)))\right] + \delta\omega\,(a_2-a_1)}{(q^{FI}(a_2) - q^P(a_1,a_2,q^{FI}(a_2)))} \tag{A.6}$$

Then, as we decrease ψ below ψ^*, the incentive-compatibility constraint starts binding and $q(a_2)$ adjusts. Then, the optimal market tightness for sellers of quality a_2 is determined by

$$\begin{aligned}&p(\theta^{FI}(a_1))\left(q^{FI}(a_1) - \beta\nu^{\mathrm{s}}(a_1) + \delta\omega a_1\right) \\ &= p(\theta(a_2))\left((1-\psi)q(a_2) + \psi q^P(a_1,a_2,q(a_2)) - \beta\nu^{\mathrm{s}}(a_1) + \delta\omega a_1\right).\end{aligned}$$

Replacing the price $q(a_2)$ from the buyer's indifference condition, we obtain

$$\begin{aligned}&p(\theta^{FI}(a_1))\left(q^{FI}(a_1) - \beta\nu^{S}(a_1) + \delta\omega a_1\right) \\ &= p(\theta(a_2))(1-\psi)\nu^b(a_2) - (1-\psi)\theta(a_2)\chi + p(\theta(a_2))\left(\psi q^P(a_1,a_2,q(a_2)) - \beta\nu^S(a_1) + \delta\omega a_1\right).\end{aligned}$$

Comparative statics. Let us differentiate the constraint with respect to ψ in the region where the incentive-compatibility constraint binds (i.e., $\psi \le \psi^*$). Then,

$$\frac{d\log\theta(a_2)}{d\psi} = \frac{\left(q(a_2) - q^P(a_1,a_2,q(a_2))\right)}{((1-\psi)q(a_2) + (\psi q^P(a_2,a_1,q(a_2)) - \beta\nu^{\mathrm{s}}(a_1) + \delta\omega a_1))} > 0$$

where the last inequality follows from $q(a_2) \ge q^{FI}(a_2) > q^{FI}(a_1) \ge q^P(a_1,a_2,q(a_2))$. Thus, the optimal market tightness for seller's of quality a_2 is increasing in the informativeness of the inspection.

There exists a threshold $\underline{\psi}$ such that continuation values are increasing for $\psi \ge \underline{\psi}$. The last item that we need to check is that $\bar{v}(a_2) > \bar{v}(a_1)$ in the case where the incentive-compatibility constraint is binding so that we can guarantee the existence of the equilibrium.

Using the incentive-compatibility constraint we have

$$\nu^S(a_1) - \bar{v}(a_1) = \nu^{\mathrm{s}}(a_2) - \bar{v}(a_2) + p(\theta(a_2))[\psi(q^P(a_1, a_2, q(a_2)) - q(a_2) + \bar{v}(a_2) - \bar{v}(a_1)].$$

Using $q(a_2) > q^{FI}(a_2)$, $q^P(a_1, a_2, q(a_2)) = q^B(a_2, a_1)$, this constraint can be written as

$$(1 - \beta(1 - p(\theta_2)))(\bar{v}(a_2) - \bar{v}(a_1)) = \beta\psi p(\theta_2)[q(a_2) - q^B(a_2, a_1)] - \delta\omega(a_2 - a_1).$$

Hence, $\bar{v}(a_2) \geq \bar{v}(a_1) \geq 0 \iff \beta\psi p(\theta_2)[q(a_2) - q^B(a_2, a_1)] \geq \delta\omega(a_2 - a_1)$. Replacing $q(a_2)$ from the buyer's indifference condition we obtain

$$\bar{v}(a_2) \geq \bar{v}(a_1) \geq 0 \iff \beta\psi[p(\theta_2)[\nu^{\mathrm{b}}(a_2) - q^B(a_2, a_1)] - \chi\theta_2] \geq \delta\omega(a_2 - a_1)$$

The function of θ_2 on the left-hand side is strictly concave. Since we are constrained on the space $q(a_2) > q^{FI}(a_2)$ and continuation values are increasing under full information, we know that one of the solutions to

$$\beta\underline{\psi}\left[p(\theta_2)[\nu^{\mathrm{b}}(a_2) - q^B(a_2, a_1)] - \chi\theta_2\right] = \delta\omega(a_2 - a_1)$$

is greater than ψ^*. Hence, the above equation has a unique solution on the interval $[0, \psi^*]$, which defines the threshold $\underline{\psi}$ such that continuation values are increasing. Notice that when $\delta \to 0$, we have that $\underline{\psi} \to 0$.

There is no pooling equilibrium in the two-quality case. Using our earlier proof that rules out pooling equilibria, we need the condition that for all $\omega, \hat{a}, q \in \mathcal{A}^2 \times \mathbb{R}+$, $\psi q^P(\hat{a}, \omega, a, q) - \nu^S_{t+1}(\omega, a) + \delta\omega a$ is monotonous in a. Here, this condition is trivially satisfied. Indeed the function is always monotonous over the set of unobserved qualities as there are only two (either one is greater than the other, or the reverse). ■

A.2 Additional results

A.2.1 Bargaining as a special case of inspection-adjusted price function

Let us now use a form for q^P that is consistent with the properties we have assumed. The quality has been revealed by the inspection on submarket $(\hat{a}, q)$. The outside option of the buyer is 0 , while the outside option of the seller is $\Lambda\nu^S_t(\omega, a) - \delta\omega a$, which corresponds to its continuation value if the transaction fails. The surplus of the transaction is :

$$S_t(\hat{a},\omega,a,q) = \nu_t^b(\omega,a) - \left[\Lambda_t \nu_{t+1}^S(\omega,a) - \delta\omega a\right]$$

We assume that the bargaining power of the seller is $\phi < \eta$ and obtain :

$$q^P(\omega,a,\hat{a},q) = min(\phi\nu_t^b(\omega,a) + (1-\phi)\left[\Lambda_t\nu_{t+1}^S(\omega,a) - \delta\omega a\right], q) \tag{A.7}$$

Let us now check the assumptions used in the proofs of the propositions.

We see that for two types $a' > a$ on this submarket :

$$q^P(\omega,a',\hat{a},q) - q^B(\omega,a',\hat{a},q) = \phi(\nu_t^b(\omega,a') - \nu_t^b(\omega,a)) + (1-\phi)\left[\Lambda_t(\nu_{t+1}^S(\omega,a') - \Lambda\nu_{t+1}^S(\omega,a)) - \delta\omega(a'-a)\right] \geq 0$$

Since $\phi \leq \eta$, we also have $q^P(\omega,a,\hat{a},q) \leq q^{FI}(a)$.

This also immediately yields that $\nu_t^b(\omega,a) - \psi q^P(\omega,a,\hat{a})$ is increasing in a for all $\omega \in \Omega$ and $\hat{a} \in \mathcal{A}$ as q^P does not depend on the signal and $\nu_t^b(\omega,a) - \psi q^P(\omega,a) = (1-\phi\psi)\nu_t^b(\omega,a) + (1-\phi)(\Lambda_{t,t+1}\nu_{t+1}^S(\omega,a) - \delta\omega a)$.

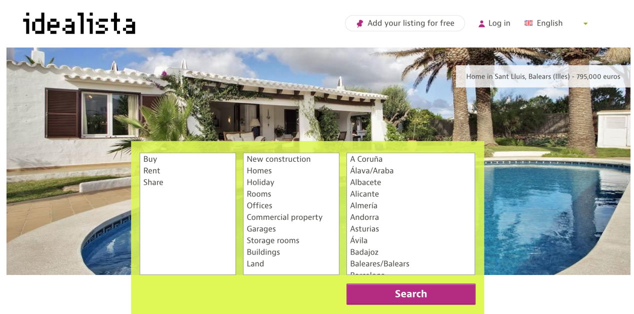
We also obtain $\psi q^P(\hat{a},\omega,a) - (\Lambda_{t,t+1}\nu_{t+1}^S(\omega,a) - \delta\omega a) = \phi\nu_t^b(\omega,a) - \phi\left[\Lambda_t\nu_{t+1}^S(\omega,a) - \delta\omega a\right]$ is increasing in a, yielding in particular the result that there is no pooling equilibrium. Finally we have $q^P(\hat{a},\omega,a,q) \in [min(q,\nu^b(\omega,a)), min(q,\nu^S(\omega,a))]$ which satisfies our assumption that there is always a transaction as long as the surplus is positive.

B Empirical Appendix

B.1 The online platform

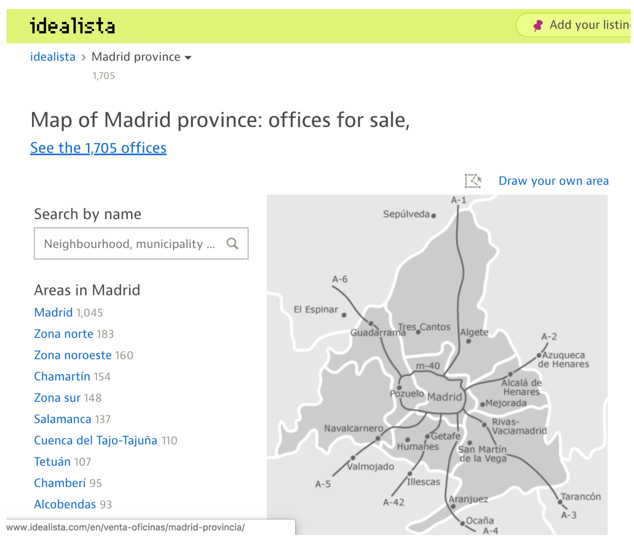
This subsection describes how the platform works. When entering the website, the buyer encounters the screen shown in Figure 2. The platform asks the client to choose a type of transaction (buy, rent, or find a shared space), the type of property (retail store, office, etc.), and the location.

FIGURE 2: Main Website



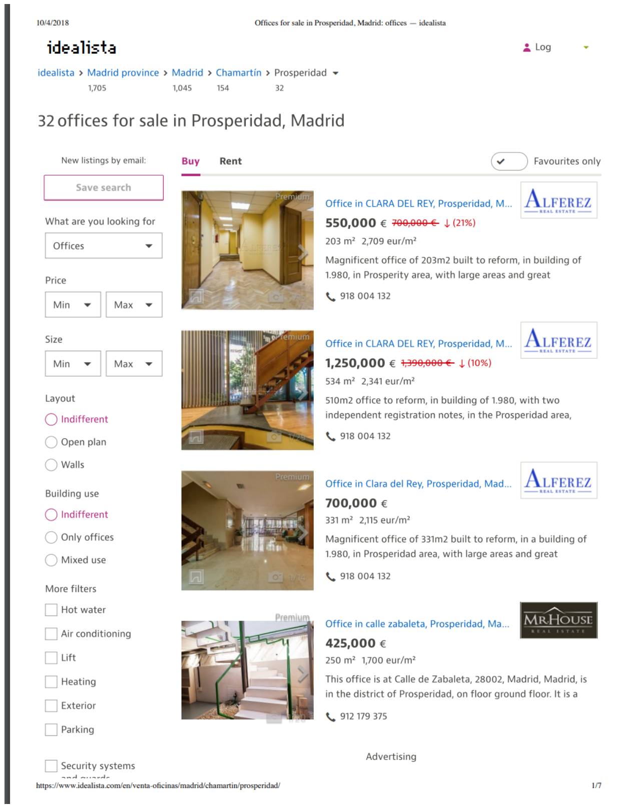
Once those options are selected, suppose the client wants to find a unit in Madrid (Figure 3). There, the website shows the number of properties available for sale by area in the city.

FIGURE 3: Options Madrid



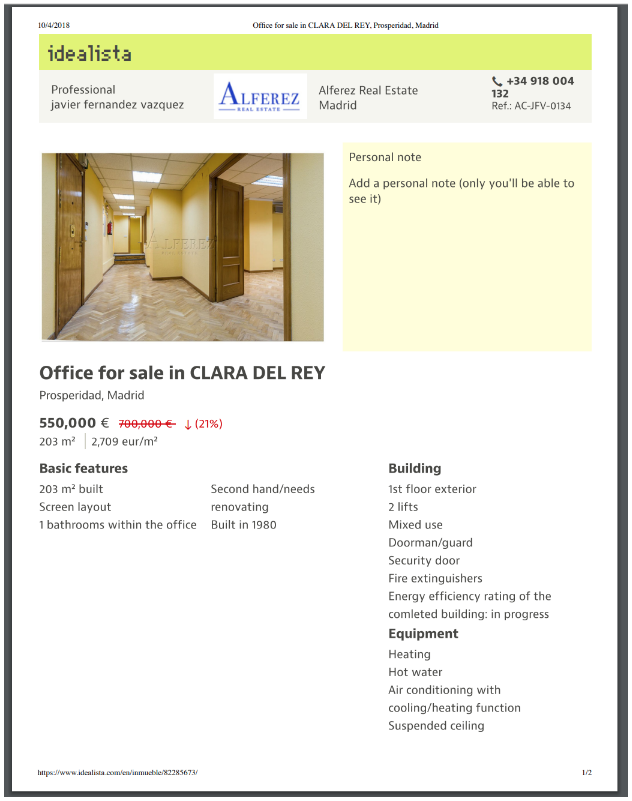
After choosing a narrower location within the city (not shown here), the client finds a scrolling list of the available units that meet her requirements, as shown in Figure (4). There, the user can include more filters depending on her requirements for layout and amenities.

Figure 4: Available listings in a narrow location in Madrid



When the user finds a unit that may be to her taste and clicks on it, a window pops up with the details shown in Figure 5 plus text details not shown here. The main information the listing contains is the unit description with pictures, price, change in price, area, date of construction, and other amenities and equipment.

FIGURE 5: A listing on the website



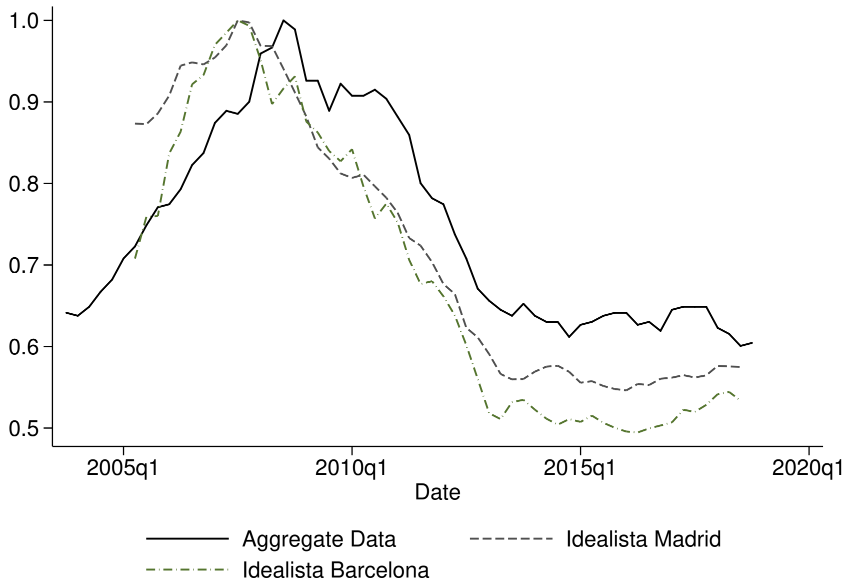
B.2 Representativeness of the dataset

In this subsection, we analyze the representativeness of the dataset, showing that our data is consistent with aggregate patterns observed in Spain over this period. We provide two pieces of evidence about our data. First, we show that in our data, the price index exhibits the patterns of aggregate data. Second, we show that the patterns of sales follow those of aggregate sales of structures in Spain.

Figure 6 shows the index of listed prices for properties for sale in our sample and the index of transacted prices of retail space in Spain (the latter come from official transaction records). Both indexes are normalized to one at their respective peak. We highlight the fact that the fall in prices we observe is consistent, and very similar in size to that observed for retail space in Spain during the recent financial crisis. Moreover, our index leads the aggregate index, which is expected since our index consists of listed prices. This is expected, since our index consists of listed prices and it will take properties some months to exit the database, be registered as sales, and be recorded in national statistics.

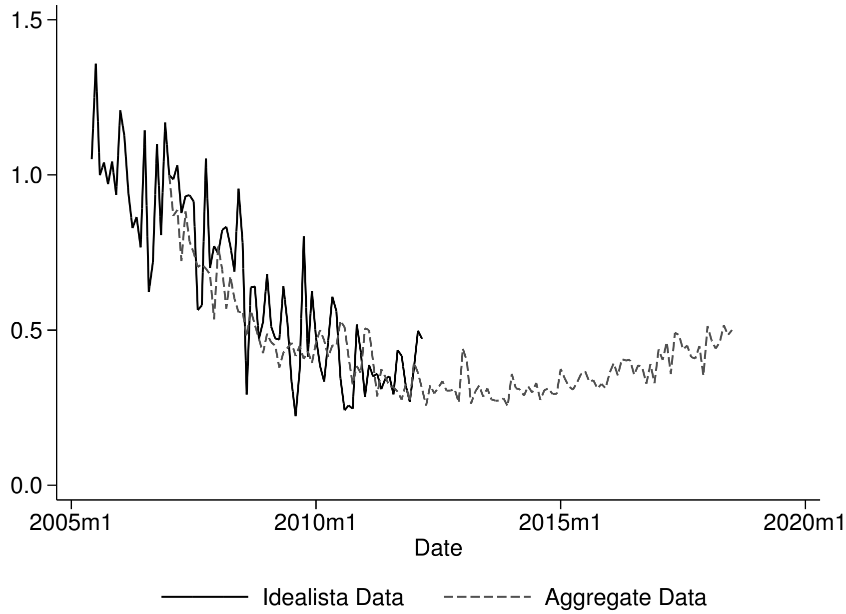
Figure 7 shows the index of sales for properties for sale in our sample and the aggregate sales index of real estate in Spain. Both indexes are normalized to take the value of 1 in the first month of 2007. The index for our data is constructed by computing the share of units that exit the database with respect to the number of active posts in that month. In the case of the aggregate number, we normalize the number of sale transactions recorded by the Statistical Agency. In doing this, we assume that the total stock of units during this period is fairly constant (we do not have information on the size of the stock). Although our index is noisier than the national estimates, the patterns of the two series are close to each other.

FIGURE 6: Price Index: Dataset versus Aggregate Data



Note: The solid line shows the price index for properties for sale in Barcelona and Madrid in our dataset. The dashed line shows the aggregate retail space price index gathered from the National Registry of Property (Registradores de España). All indices are normalized to their respective peak.

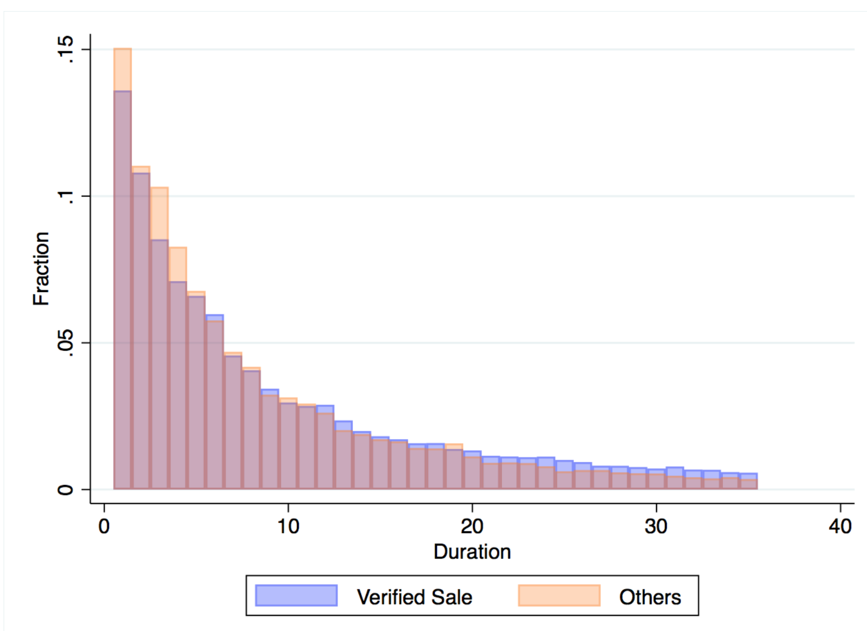
FIGURE 7: Sales Rate: Dataset versus Aggregate Data



Note: The solid line shows the sales rate for properties in our dataset. The dashed line shows the aggregate sales index of real estate gathered from the Statistical Agency of Spain (INE). Both indices take the value of one in January 2007.

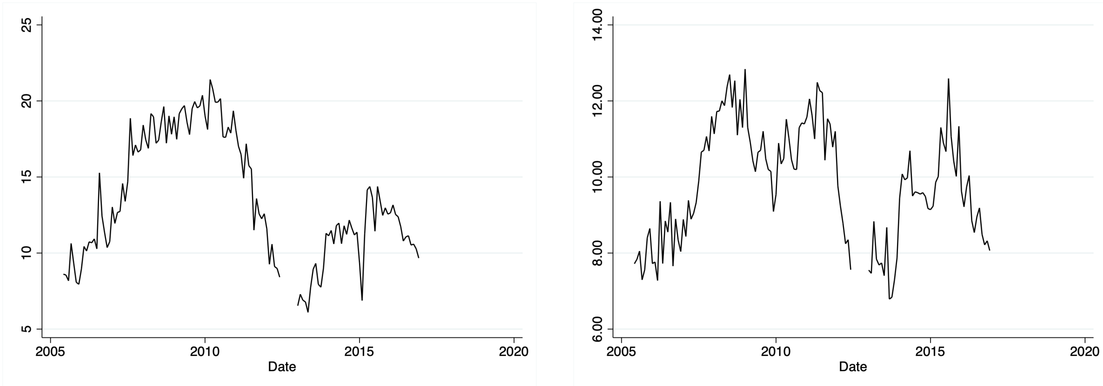
B.3 Additional Figures and Tables

FIGURE 8: Distribution of Duration: Confirmed Sales



Note: This figure compares the histogram of duration for two subgroups of listings: those that, after removing the listing from the platform, explained that they did so because the property was rented out or sold, and those that did not provide an explanation.

Figure 9: Evolution of Average Duration

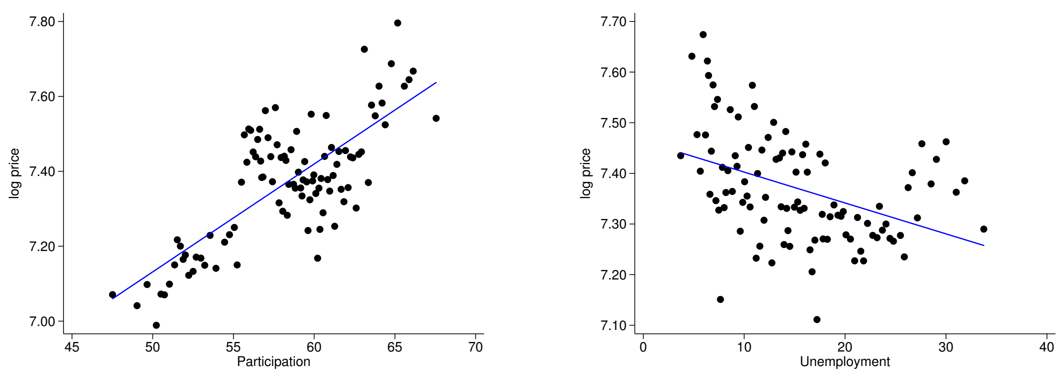


(a) Capital for Sale

(b) Capital for Rent

Note: The left panel shows the evolution of mean time to sell (in months) at monthly frequency from 2006 to 2017. The right panel shows an equivalent index for rental units. Time to sell is measured as the time difference between the entry and exit dates of each listing. Each observation contains the average time to sell for listings that entered the online platform in a given month.

Figure 10: Capital Prices and Regional Business Cycles



(a) Price and Labor-Force Participation

(b) Prices and Unemployment

Note: This figure shows the relationship between log prices of posts for sale in a given province in a given quarter year with economic variables, in this case labor-force participation and unemployment rate. The figure presents a binned scatterplot, in which we choose 100 quantiles of the relationship between the economic variable of interest and log prices and compute the average for observations in that quantile.

Table 2: Price Variation Accounted for by Listed Characteristics in New Entrants

	Sale		Rent	
Statistic	IQR	R^2	IQR	R^2
Raw Data	0.666	0.000	0.802	0.000
Benchmark	0.284	0.776	0.198	0.845
Property Fixed Effect	0.119	0.946	0.116	0.937

Note: This table extends Table (2) by including a property fixed effect, which gathers inference from properties that change their prices while they are active in the dataset. We find that after including property fixed effects, non-parametrically absorbing all of the property's time-invariant price determinants, the IQR is roughly 11% and the R^2 is roughly 0.94.

Table 1: Frequency of Price Changes for Capital

	Rent Office	Sale Office	Rent Warehouse	Sale Warehouse
Frequency of Price Changes	0.07	0.07	0.05	0.07
Frequency of Price Increases	0.02	0.02	0.02	0.02
Frequency of Price Decreases	0.05	0.05	0.04	0.05
Absolute Size of Price Changes	0.15	0.12	0.16	0.15
Absolute Size of Price Increases	0.19	0.15	0.19	0.18
Absolute Size of Price Decreases	0.14	0.11	0.15	0.14

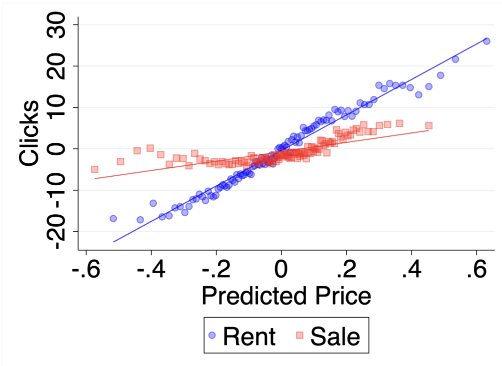
Note: This table presents price adjustment statistics by property type and operation. In order to compute the table, we first compute statistics about price changes within each property and then take averages across properties in a given time period. Finally, we compute the average over time. The first row shows the frequency of price changes, which is the average share of properties that exhibit a price change in a given month. The following two rows show the share of listings with price increases and decreases. The absolute size of price changes is computed as the absolute value of the log difference in prices over consecutive months (ignoring the zeros).

Table 3: Regression of Prices on Duration - Rent

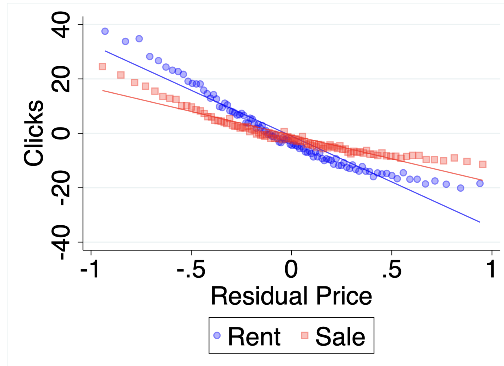
	(1) log Dur b/se	(2) log Dur b/se	(3) log Dur b/se	(4) log Dur b/se
log price	-0.092***			0.027***
	(0.001)			(0.004)
Predicted Price		-0.170***	-0.174***	
		(0.008)	(0.006)	
Residual Price		0.030***	0.030***	
		(0.004)	(0.004)	
Constant	1.848***	1.838***	1.838***	1.474***
	(0.001)	(0.001)	(0.004)	(0.018)
Observations	696874	680553	680553	686612
R^2	0.007	0.159	0.014	0.177
Subsample	Rent	Rent	Rent	Rent
Fixed Effects	No	Yes	No	Yes

Note: This table presents the results of a regression of log duration on the two components of prices, residual and predicted prices. The sample includes listings for rent only. Column 2 regresses duration on predicted and residual prices. Column 3 additionally excludes location×time fixed effects. *, **, and *** represent statistical significance at the 10%, 5%, and 1% level, respectively.

Figure 11: Relationship between Prices and Clicks



(a) Predicted Prices



(b) Residual Prices

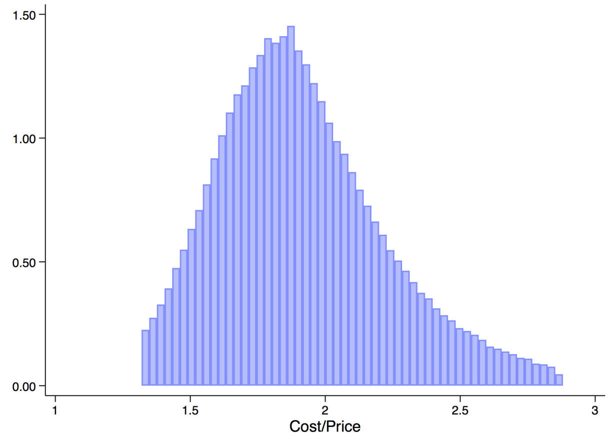
Note: Panel (a) shows the relationship between predicted prices and average monthly clicks. Panel (b) shows the relationship between residual prices and average monthly clicks. Price residuals and predicted prices are obtained after running a regression of log prices on a set of fixed effects and observable characteristics. The figures show a binned scatter plot of each relationship, after controlling for location-time-type (offices, retail space, and warehouses) fixed effects.

Table 4: Prices and Clicks

	(1) log clicks b/se	(2) log clicks b/se	(3) log clicks b/se
log price	-4.529***		
	(0.109)		
Predicted Price		8.301***	4.655***
		(0.659)	(0.796)
Residual Price		-19.107***	-19.107***
		(0.540)	(0.538)
Constant	66.538***	5.641*	23.185***
	(0.531)	(3.169)	(3.541)
Observations	448544	431966	432091
R^2	0.004	0.342	0.028
Subsample	Sale	Sale	Sale
Fixed Effects	No	Yes	Yes

Note: This table presents the results of a regression of log average monthly clicks on the two components of prices, residual prices and predicted prices. The right-hand-side variable to be the mean price over the lifetime of the listing. The first column shows a regression of log clicks on log prices. Column 2 regresses log clicks on predicted prices and residual prices, and location×time× fixed effects. Column 3 does not include fixed effects. *, **, and *** represent statistical significance at the 10%, 5%, and 1% level, respectively.

Figure 12: Required Holding Costs



C Quantitative Analysis

C.1 Model Extensions