Spatial Consumption Risk Sharing

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Abstract

This paper examines how frictions in bilateral economic linkages shape the consumption pattern across economies. Using state-level data from the US, we find that the degree of bilateral consumption risk sharing across states decreases in geographic distance. To explain this novel fact, we develop a DSGE model that incorporates trade, migration, and finance as channels of risk sharing which are subject to frictions that covary with distance. Calibrated to the US data, the model not only enables us to quantify the magnitude of the frictions in each channel, but also allows us to examine the interplay among the channels and disentangle their effects on the level, volatility, and comovement of consumption across states. Counterfactual analyses based on the model provide guidance for the design of macroeconomic policies that aim to reduce cross-region consumption disparity.

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1 Introduction

Consumption risk sharing allows different agents to experience welfare gains by reducing consumption fluctuations caused by idiosyncratic income shocks. However, frictions in economic exchanges across regions impede consumption from being smoothed across space and time. This paper explores the patterns and determinants of consumption risk sharing by exploiting the variations in bilateral economic linkages shaped by geography.

What drives imperfect consumption correlations across economies remains a central question of interest in macroeconomics since the phenomenon attests to the failure of complete markets. For example, Obstfeld and Rogoff (2000) consider the low cross-country consumption comovement as one of the major puzzles in international macroeconomics. Besides trade costs in the goods market discussed by these authors, migration costs in the labor market, and asset transaction costs in the financial market, potentially affect risk sharing since they pose barriers for economic resources to be freely mobile across economies in the presence of local shocks. In contrast with most existing literature that examines the influence of one friction, this paper extends the workhorse open economy real business cycle (RBC) model developed by Backus et al. (1992) (BKK) into a unified framework with trade, migration, and finance as channels of risk sharing. This framework enables us to disentangle the effects of frictions in different channels.

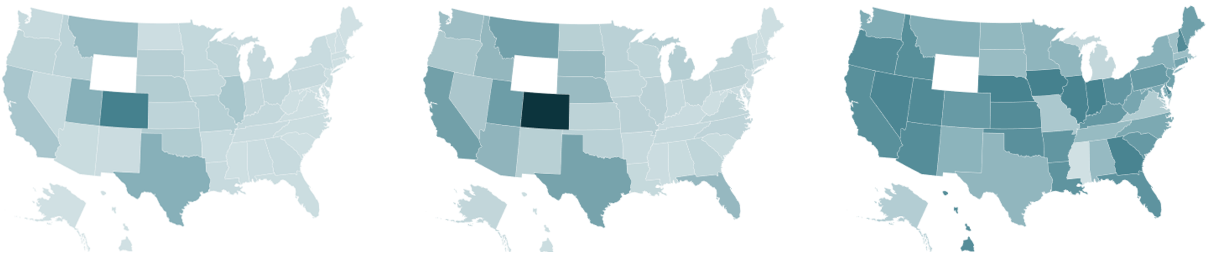
Another distinct feature of this paper is that we add a geographic dimension to our macroeconomic analysis. One similarity of the three channels of risk sharing lies in the fact that economic linkages in these channels covary with geographic distance between a pair of economies, as is documented in the literature on the gravity model of trade, finance, and migration.[1] Since these channels are important drivers for cross-economy synchronization, bilateral consumption comovement is also expected to exhibit similar geographic characteristics. To exemplify such patterns, we plot the bilateral economic ties between Wyoming and other states in the US in figure 1 and confirm that ties are generally stronger for neighboring states.[2] To capture these spatial characteristics, we embed bilateral linkages through channels of consumption risk sharing in a multi-region theoretical framework. Compared to a symmetric two-economy model such as BKK, this

[1]For example, Anderson and Van Wincoop (2003) develop a theory-grounded econometric model to characterize bilateral trade flows across countries. Portes and Rey (2005) document that bilateral equity flows decrease with distance between country-pairs. Lewer and Van den Berg (2008) develop and test a gravity model of immigration among OECD countries.

[2]Detailed data description can be found in Appendix B. Cross-state trade data are sourced from the CFS, migration data are from the IRS, and consumption data are from the BEA. Comprehensive data for state-to-state financial flows are not available to our knowledge.

multi-region framework allows us to examine the aggregate influences across bilateral exchanges on each economy's consumption. Compared to the quantitative spatial models surveyed by Redding and Rossi-Hansberg (2017), this RBC framework has the advantage of examining the second moments (variance and covariance) besides the first moment (level) of macroeconomic fundamentals, both of which are essential for welfare analysis.

Figure 1: Wyoming's Bilateral Ties with Other States



(a) Bilateral Trade (b) Bilateral Migration (c) Consumption Corr.

This figure plots the economic linkages between Wyoming (in white) and other states in the U.S. averaged over the sample period of 1997-2017. A darker color suggests a higher value of trade and migration flows (the sum of inflows and outflows) as well as a greater correlation coefficient of real consumption per capita.

We focus on state-level analysis within the US, but the general framework can be easily tailored to other settings of interest.[3] In the empirical section, we explore the relationship between consumption risk sharing and geographic distance. Following the macroeconomic literature such as Asdrubali et al. (1996) and Kose et al. (2009), we measure a region's consumption risk sharing as the response of its relative consumption growth to its relative output growth. A greater response suggests a lower degree of consumption risk sharing, since the region's own income predominantly drives its consumption fluctuations. We first calculate the degree of bilateral risk sharing using the output and consumption per capita data of the fifty US states over the period 1977-2019. In the next step we document that risk sharing is weaker for state pairs that are more geographically distant: Every 1% increase in distance lowers the response of relative consumption to output growth between a pair of states by 0.151 (or 0.402 standard deviations). This spatial characteristic of bilateral economic linkages echoes the prediction of the classic gravity model. As a novel empirical regularity for consumption, the finding points to the existence of barriers to risk sharing that are influenced by geography.

[3] For example, the model can be applied to intranational analysis of another country, or international analysis of the European Union which exhibits a high level of integration for goods, financial, and labor markets. Given that frictions are relatively low across states in the US, our estimates provide a lower bound on the frictions' importance.

As an event study, we examine the 2006 North Dakota (ND) oil boom to verify the importance of geography in driving the variation of consumption gains across states. Through panel regressions, we find that due to the positive output shock, bilateral linkages of ND with other states exhibit strong geographic patterns: ND witnessed greater migration and trade inflows from states located in closer proximity. These states also experienced stronger consumption comovements with ND following the oil shock.

Motivated by the empirical findings, we develop a DSGE model to examine the channels that may shape this geographic pattern of consumption synchronization. Our model is populated by representative households who reside in different states connected by three forms of bilateral economic exchanges: trade, migration, and finance. In the trade channel, we follow the classic Armington (1969) model and assume each state produces one type of intermediate good which is traded across states subject to iceberg trade costs. In the migration channel, we follow the framework developed by Artuc et al. (2010) with modifications and assume households make forward-looking migration decisions in response to consumption differentials across states under migration frictions. Both the trade and migration models mentioned above have been adopted in the recent literature that examines the macroeconomic impacts of economic linkages (see, for example, Caliendo et al. (2018), House et al. (2018), and House et al. (2020)).

What is more unique about our spatial analysis is the modeling of financial flows. Due to the difficulty of incorporating a frictional financial channel in a multilateral model, the existing literature has either focused on extreme scenarios (autarky or complete markets) or taken asset positions from the data as exogenous. In contrast to these approaches, we set up a portfolio choice problem and endogenize households' preferences among assets driven by their risk sharing needs. Furthermore, we introduce bilateral financial frictions as iceberg transaction costs on asset returns following Heathcote and Perri (2013) and Tille and Van Wincoop (2010).[4] To derive portfolios under frictions, we employ Devereux and Sutherland (2011)'s solution technique, which combines a second-order approximation of the Euler equation and a first-order approximation of other equations to derive the steady-state portfolio in a DSGE model. The portfolio choice will in turn affect consumption correlations, which allows us to quantify both the magnitude of bilateral financial frictions and the distortion of consumption caused by them.

[4]Financial frictions can take alternative forms to asset transaction costs. For example, Okawa and Van Wincoop (2012) discuss the comparability of information frictions and transaction costs in terms of their prediction for the gravity model of financial flows. Even within a country, there exist such financial frictions that covary with geography. Empirical evidence for this is the 'home bias at home' phenomenon documented by Coval and Moskowitz (1999).

To illustrate the mechanism of how the three channels jointly shape consumption synchronization, we start with a two-economy framework à la BKK. By conducting comparative analyses, we find that the interplay among the three channels of risk sharing yields non-monotonic predictions of how the various frictions affect consumption correlations across states. For example, higher financial frictions, by tilting portfolios towards domestic assets, reduce bilateral consumption correlations in general, consistent with the argument from the neoclassical model of risk sharing (Lucas (1982)). Nevertheless, when financial frictions are so high as to deteriorate wealth accumulation, population moves out of the region which has experienced a positive productivity shock. Meanwhile, the productivity shock leads to a depreciation of the region's terms-of-trade on impact which translates into lower wage rates. Therefore, these migration outflows which raise local wages due to decreased labor supply, will stabilize the cross-region wage disparity and lead to stronger consumption comovement. This analysis, by showing the effects of the channels' interactions on consumption, underscores the importance of examining these channels in an integrated general equilibrium setting.

We then extend the two-region to a multi-region framework for a quantitative assessment of the theory. In this analysis where we still focus on the bilateral linkages between a pair of states, we also consider the rest of the economy (ROE) which exerts 'multilateral resistance' on the state-pair under examination in the spirit of Anderson and Van Wincoop (2003). Specifically, we develop a trilateral framework that consists of the state-pair and ROE which aggregates all the other states from the state-pair's perspective. This trilateral framework allows us to overcome the computational challenge of solving the portfolio choice problem in a DSGE model with many economies of unequal sizes. For parametrization of the quantitative model, we calibrate trade and migration frictions to match a state-pair's bilateral trade and migration flows. Furthermore, we use the state-pair's coefficient of risk sharing estimated from the empirical section as a targeted moment to solve for the portfolio that supports this consumption comovement, and then recover the bilateral financial frictions from this specific portfolio arrangement. We conduct the estimation for all the state pairs and confirm the geographic feature of bilateral frictions: For a 1% increase in distance, trade, migration, and financial frictions increase by 0.53%, 0.10%, and 0.23% respectively.

We proceed to quantify the impacts of frictions on consumption through a series of counterfactual analyses where frictions are turned off. When evaluating the steady state level of consumption, we find that the reduction in trade costs benefits almost all the states, whereas the reduction in migration costs generates disparate predictions for different states. The most affluent states such as New York and California benefit from population inflows, while other states witness lower wage income under labor market integration. In terms of second moments, eliminating three types of frictions uniformly leads to lower consumption volatility in general. The mean reduction in consumption volatility across states is 0.7%, 1.0%, and 0.3% respectively when bilateral trade, migration, and financial frictions are turned off. This result supports the argument that reducing barriers to risk sharing yields welfare gains by smoothing consumption fluctuations. These counterfactual analyses not only disentangle the influences of each channel on the level and volatility of consumption, but also provide guidance for fiscal policies which, by mitigating the impacts of the frictions, reduce consumption inequality. Using an example that studies the direction and magnitude of transfers across states to alleviate the effects of trade costs on the level of consumption, we show that our framework offers a useful tool for the design of macroeconomic policies which aim to narrow consumption disparity across space and time.

This paper contributes to the macroeconomic literature on consumption risk sharing by adding the geographic dimension, which enriches the understanding of the patterns and determinants of consumption comovement across economies. To explain the failure of consumption risk sharing, the existing international macroeconomic literature examines frictions in the financial market (e.g. Cole and Obstfeld (1991), Baxter and Crucini (1995), Kollmann (1995), and Lewis (1996)) and in the goods market (e.g. Dumas and Uppal (2001), Corsetti et al. (2008), and Eaton et al. (2016)) that impair consumption smoothing across countries. Nevertheless, many of these works focus on one channel only and therefore do not analyze multiple channels whose interactions are essential for the consumption pattern. Furthermore, most papers employ a two-country framework, which is not ideal to study the aggregate influences of bilateral linkages, with potential substitutability and complementarity, on macroeconomic fundamentals. There are two notable exceptions that are closer to our work. First, Fitzgerald (2012) disentangles the impacts of trade costs and financial frictions (measured as the departure of countries' relative consumption to a benchmark country from the consumption predicted by complete markets) on cross-country risk sharing. Second, House et al. (2018) combine frictional trade and migration channels in a multi-region framework to quantify the benefits of labor mobility in the European Union. Compared to these two papers, our portfolio choice framework makes it possible to quantify financial frictions at the bilateral level for cross-sectional comparison and counterfactual analysis. These bilateral financial frictions are important in shaping the variation in bilateral consumption comovement in our risk-sharing analysis.

In the domestic context, Asdrubali et al. (1996), Hess and Shin (1998), Crucini (1999), Athanasoulis and Van Wincoop (2001), Del Negro (2002), and Kalemli-Ozcan et al. (2010) pioneered the work on consumption risk sharing within the US. These empirical works quantify the level of intranational risk sharing using state-level data. At the micro level, seminal papers including Storesletten et al. (2004) and Heathcote et al. (2014) explore heterogeneity across the US households in terms of the impacts of income on consumption. Neither these macro nor micro perspectives focus on the effects of bilateral economic linkages across regions or the influences of region-specific conditions on households' consumption and migration decisions. Therefore, our paper extends this literature by considering additional channels for facilitating consumption smoothing within a country.

Our paper is also influenced by recent developments in the spatial economics literature. As is discussed in the comprehensive survey by Redding and Rossi-Hansberg (2017), new quantitative models of economic geography provide powerful yet tractable tools to characterize the distribution of economic activity across a large number of locations of uneven sizes. There are two dimensions along which our work differs from and potentially contributes to that strand of literature. First, we add a financial channel under bilateral frictions by setting up the portfolio choice framework, which complements existing papers that primarily focus on the real side of the economy and linkages in the goods and labor markets. Second, our framework has the advantage of examining the correlation and volatility in addition to the level of consumption under endogenous financial investment. Admittedly, the local solution method used in this RBC framework is not as flexible as the global method used in the quantitative trade literature, yet it proposes a new technique to incorporate a frictional financial channel in a multi-region framework.

Lastly, this paper contributes to the extensive empirical literature on the gravity model. Since being introduced by Isard (1954) and Tinbergen (1962), the model has emerged as a classic framework in the trade literature. More recently, seminal works including Anderson and Van Wincoop (2003) and Eaton and Kortum (2002) refine the theoretical foundations of the framework that rationalize empirical regularities of bilateral trade. In addition to trade, the gravity model has been applied to a wide range of topics including financial assets (e.g. Portes and Rey (2005), Martin and Rey (2004), Aviat and Coeurdacier (2007), and Okawa and Van Wincoop (2012)) and population flows (e.g. Lewer and Van den Berg (2008) and Ramos and Suriñach (2017)). Nevertheless, less is known about the effects of distance on macroeconomic fundamentals. Our paper, together with Chertman et al. (2020) for cross-country analysis, adds to this literature by exploring the role of geographic distance in shaping the consumption patterns.

2 Empirical Motivation

This section empirically establishes the importance of geographic distance for bilateral consumption risk sharing by using the US data. Our analysis consists of two parts. First, we use the state-level consumption and output data to compute the degree of bilateral consumption risk sharing and find that it weakens with the geographic distance between state pairs. Second, we examine the 2006 North Dakota oil shock as an event study to verify the role of geography in shaping the variation in consumption comovement.

We measure a region's consumption risk sharing as the response of its relative consumption growth to its relative output growth following the macroeconomic literature such as Asdrubali et al. (1996) and Kose et al. (2009). In particular, we focus on bilateral risk sharing so that we can exploit pair-specific factors including geographic distance in order to examine the patterns and determinants of consumption comvement across regions. Specifically, we evaluate risk sharing between state i and j from

$$\Delta \log\ c_{it} - \Delta \log\ c_{jt} = \alpha_{ij} + \beta_{ij}(\Delta \log\ y_{it} - \Delta \log\ y_{jt}) + \epsilon_{ijt}, \tag{1}$$

where $\Delta \log\ c_{it}$ ($\Delta \log\ c_{jt}$) and $\Delta \log\ y_{it}$ ($\Delta \log\ y_{jt}$) denote the growth of log real per-capita consumption and output of state $i(j)$ at time t. The coefficient β_{ij} measures the degree of bilateral consumption risk sharing. In the case with perfect risk sharing, relative consumption growth equals zero regardless of relative output growth, which yields a coefficient of 0. In the opposite case with complete autarky, a state's consumption is solely determined by its own output, which implies a coefficient of 1. Therefore, a lower value for the coefficient β_{ij} suggests a higher degree of bilateral risk sharing.

The data using which we evaluate equation 1 are obtained from the following sources. The US Bureau of Economic Analysis (BEA) reports real gross state product (GSP) from 1977 to 2019. State-level consumption data from the BEA have short coverage (from 1997 onwards), which is not ideal for our analysis of risk sharing that requires long-horizon data. Therefore, we follow Asdrubali et al. (1996)'s method of constructing state-level consumption by rescaling state-level retail sales by the country-level ratio of private consumption to retail sales, both of which are available from the BEA. Moreover, we use Nakamura and Steinsson (2014)'s state-level inflation series to convert nominal to real consumption. Appendix B provides the details of these datasets and describes the method we use to analyze the data.

Panel A of table 1 presents bilateral correlations among all the state pairs over the sample period. The correlations are calculated using the HP-filtered real consumption and output per capita (in logs). The mean bilateral output correlation is 0.422 which is higher than the consumption correlation 0.340. This stylized fact across states is consistent with the international evidence documented by Lewis (1996). Since this empirical regularity contradicts the theoretical prediction in complete markets, it remains a perplexing puzzle in international macroeconomics (Obstfeld and Rogoff (2000)). In this paper we use domestic data to explore the patterns and determinants of risk sharing, which potentially sheds light on the puzzle in the international context.

We establish an empirical gravity model of consumption risk sharing by deriving a cross-sectional prediction for consumption comovement across states. In particular, we explore the implications of geographic distance for bilateral consumption risk sharing by conducting a two-stage regression. In the first stage, we follow equation 1 to estimate the bilateral risk-sharing coefficients for all the state pairs over the sample period. Panel B of table 1 summarizes the statistics of the estimated coefficients $\hat{\beta}_{ij}$. The mean and median values are 0.515 and 0.501 respectively. The fact that $\hat{\beta}_{ij}$ is between 0 and 1 implies imperfect cross-state consumption risk sharing.

In the second stage, we regress the estimated $\hat{\beta}_{ij}$ on the log of geographic distance:

$$\hat{\beta}_{ij} = \alpha + \gamma \log(dist_{ij}) + \Gamma X_{ij} + \nu_{ij}. \tag{2}$$

Our hypothesis is that state pairs with greater geographic distance exhibit weaker consumption risk sharing, since bilateral economic exchanges which facilitate consumption comovements potentially face frictions that increase with bilateral distance. γ in equation 2 is therefore expected to be positive under this hypothesis. When constructing the cross-state geographic distance, we apply the Haversine formula to state capitals' longitude and latitude to approximate the distance between two states. In addition, we use the shipment distance from the Commodity Flow Survey (CFS) and verify the robustness of our empirical findings (shown in table A.2).[5] The results reported in table 2 confirm our hypothesis that bilateral geographic distance and risk-sharing coefficients are significantly and positively correlated. In column (1), when distance rises by 1%, bilateral risk sharing weakens by 0.151 (or 0.402 standard deviations). In column (2) we control for state pairs' GSP per capita averaged over the sample period and find that risk sharing is stronger for states with higher income levels. Therefore, bilateral risk sharing covaries

[5]The CFS reports the shipment mileage between origin and destination ZIP code points for commodity flows used for domestic expenditure within the US. We use the average mileage of shipments between two states to calculate this CFS-based bilateral distance.

Table 1: Summary Statistics of Output, Consumption, and Risk Sharing Coefficients

	Mean	Median	Std. Dev.	Obs.
A. Bilateral Correlation				
Output	0.422	0.479	0.316	1225
Consumption	0.340	0.388	0.329	1225
B. Risk Sharing Coefficient				
$\hat{\beta}_{ij}$	0.515	0.501	0.292	1225

Bilateral correlation of output (consumption) is calculated as the correlation of HP-filtered real output (consumption) per capita in logarithms among all the state pairs over the sample period from 1977-2019. β_{ij} is estimated as the response of the relative consumption growth to the relative output growth as specified in equation 1.

Table 2: Spatial Pattern of Risk Sharing

Dep. Var: $\hat{\beta}_{ij}$	(1)	(2)	(3)	(4)
$\log(d_{ij})$	0.151 ***	0.156 ***	0.220 ***	0.211 ***
	(0.010)	(0.010)	(0.012)	(0.012)
$\log(\bar{y}_1 \cdot \bar{y}_2)$		-0.099 ***	-0.061 *	0.052
		(0.032)	(0.035)	(0.038)
Land Area			-0.038 ***	-0.022 ***
			(0.006)	(0.006)
Mainland			0.117 ***	0.079 ***
			(0.025)	(0.024)
Coastal			0.018	0.023 *
			(0.014)	(0.014)
Contiguity			0.128 ***	0.102 ***
			(0.033)	(0.033)
# Neighboring States			-0.002	-0.005
			(0.004)	(0.004)
# MSA			0.001	-0.002 *
			(0.001)	(0.001)
# Shared MSA			0.021	0.022
			(0.023)	(0.022)
Industrial Dissimilarity (Pol_{ij})				-5.480 ***
				(0.754)
Political Dissimilarity (Ind_{ij})				0.069 **
				(0.032)
Obs.	1225	1225	1225	1225
R^2	0.161	0.169	0.255	0.288

Robust standard errors in parentheses. *** significant at 1%. The dependent variable is the risk sharing coefficient $\hat{\beta}_{ij}$, which is estimated using the real consumption and output data over 1977 – 2019. d_{ij} denotes the geographic distance between state i and j. $\bar{y}_i$ denotes the time-averaged output per capita of state i. Other control variables include a state-pair's geographic characteristics as well as political and industrial dissimilarity.

with distance and income per capita in the same direction as trade flows in the classic gravity model. In column (3) we consider other geographic variables of the state pair including the product of their land sizes in square miles (in logs), the number of mainland and coastal states,[6] a contiguity dummy which equals one for state pairs sharing borders, and the total number of neighboring states to capture the state pair's multilateral ties with adjacent states. Finally, we have the aggregate number of Metropolitan Statistical Area (MSA) and the number of MSA that geographically spans the two states under examination. The numbers of MSA matter for the percentage of commuters whose location of residence and consumption differs from location of income. After controlling for these geographic variables, we find the signs of the coefficients for distance and output per-capita to remain the same as in column (2).

Furthermore, we consider measures of political and industrial proximity across states which potentially affect risk sharing according to the literature. For example, Parsley and Popper (2021) document stark business cycle asynchronicity among blue versus red states in the US, and reason that differences in fiscal policies potentially explain how political division shapes this pattern of risk sharing. In this spirit, we construct a state's position on the political spectrum based on whether its voter chose a Republican or a Democratic candidate ($Pol_{it} = 0$ or 1) during presidential elections, and take the squared difference between the mean values over the sample period from 1976 to 2020 (denoted as $\bar{P}ol_i$) to measure the political remoteness between a pair of states

$$Pol_{ij} = (\bar{P}ol_i - \bar{P}ol_j)^2. \tag{3}$$

Meanwhile, the complementarity of industrial structures across states not only influence their output synchronization, but also potentially covary with consumption synchronization (Kalemli-Ozcan et al. (2003)). Therefore, we consider a measure of industrial remoteness by comparing sectoral composition between states

$$Ind_{ij} = \sum_{s=1}^{S} (b_{i,s} - b_{j,s})^2, \qquad \text{where} \qquad b_{i,s} = \frac{\bar{Y}_{i,s}}{\sum_{s=1}^{S} \bar{Y}_{i,s}}. \tag{4}$$

$\bar{Y}_{i,s}$ denotes the output of sector s in state i averaged over the sample period sourced from the BEA,[7] and $b_{i,s}$ hence computes the share of sector s in state i's output. By aggregating its squared difference across sectors, Ind_{ij} measures the overall dissimilarity

[6] These numbers take values 0, 1, or 2 for a pair of states. Mainland states refer to the 48 contiguous states. Coastal states refer to the states that are not landlocked and therefore have a coastline.

[7] We use the real sectoral output series (SAGDP9N) from the BEA, which reports data based on the 2012 North American Industry Classification System (NAICS) at the 3-digit level.

of industrial profiles between state i and j. Based on the results reported in column (4), state pairs with a greater political similarity and industrial dissimilarity achieve a higher level of risk sharing, consistent with the results documented by Parsley and Popper (2021) and Kalemli-Ozcan et al. (2003). Meanwhile, the coefficient of geographic distance remains to be economically and statistically significant.

In addition the baseline estimation detailed above, we perform two sets of tests to verify the robustness of our finding. First, we consider alternative data sources for state-level consumption and price levels, as well as for bilateral geographic distance. Second, we reconstruct measures of bilateral risk sharing after controlling for 1) state-level demographic variables which potentially shift aggregate demand over time including age, gender ratio, and education level, and 2) states' distinct exposure to aggregate country-level shocks. The results reported in Appendix A suggest that our finding remains robust.

After exploring the general covariance between bilateral risk sharing and geographic distance using long-term data, we conduct an event study to verify the importance of geography for bilateral economic linkages including consumption comovement. Specifically, we focus on the North Dakota oil supply shock that started from the surprising discovery of oil by a petroleum geologist in 2006. The discovery provides a natural experiment for us to evaluate the impacts of a local output boost. The rapid oil extraction since the discovery has not only fueled the economic boom of North Dakota (ND hereafter) but also positively affected other states through their economic exchanges with ND.

To establish the spatial feature of economic linkages in the wake of the oil shock, we run a panel regression with all the state pairs formed by ND over the period from 1991 to 2019 where migration and trade data are available. The regression is specified as follows

$$X_{ijt} = \alpha_0 + \alpha_1 Oil_t + \sum_{m=1}^{T} \alpha_{2m} Oil_{t-m} + \alpha_3 \log(dist_{ij}) + \sum_{n=0}^{T} \alpha_{4n} Oil_{t-n} \times \log(dist_{ij}) + \alpha_{5t} I_t + \zeta_{ijt}. \tag{5}$$

X_{ijt} represents bilateral variables of interest including migration flows (mig_{ijt}), trade values (trd_{ijt}), and relative per-capita consumption growth between state i as ND and j as any other state. For migration and trade, we focus on the log of ND's population and goods inflows from other states to capture the spillover of the positive shock. For the consumption growth, we consider both $\Delta c_{ijt} \equiv \Delta \log c_{it} - \Delta \log c_{jt}$ and $\Delta \tilde{c}_{ijt} \equiv (\Delta \log c_{it} - \Delta \log c_{jt}) - (\Delta \log y_{it} - \Delta \log y_{jt})$. The latter can be regarded as the consumption growth unexplained by the output growth of ND relative to other states, which provides a more robust measure of consumption risk sharing. To isolate the responses of these

Table 3: Bilateral Linkages after the Oil Shock

Dep. Var:	(1) $\log(mig)$	(2) $\log(trd)$	(3) Δc	(4) $\Delta\tilde{c}$
Oil_t	0.124		-0.009	0.014
	(0.465)		(0.049)	(0.054)
$\sum_{m=1}^{T} Oil_{t-m}$	-0.974	1.883 *	-0.045	0.098
	(0.599)	(0.967)	(0.077)	(0.063)
$\log(dist)$	0.013	0.012	-0.002	-0.001
	(0.014)	(0.075)	(0.002)	(0.002)
$\sum_{n=0}^{T} Oil_{t-n} \times \log(dist)$	-0.394 ***	-0.578 *	0.049 ***	0.040 **
	(0.146)	(0.325)	(0.017)	(0.017)
Observations	1,360	244	1,372	1,372
R^2	0.645	0.657	0.650	0.676

Robust standard errors in parentheses. *** significant at 1%, ** at 5%, and * at 10%. The dependent variables include North Dakota (ND)'s migration ($\log(mig)$) and trade ($\log(trd)$) inflows from other states, as well as ND's per-capita consumption growth relative to other states (Δc), and the relative consumption adjusted for output growth ($\Delta\tilde{c}$). $\log(dist)$ denotes the geographic distance between ND and other states. Oil_t is a dummy variable for the oil shock to ND in 2006. Its coefficient is missing in column (2) since trade data from the CFS are not available that year.

variables to the oil shock as deviations from their long-term trend, we take the difference between the realization of these bilateral variables at time t and their mean values over the sample period, and use these demeaned values for the dependent variables. Among the independent variables, we control for time fixed effects (denoted as I_t) which reflect the aggregate shocks that happen at time t. Furthermore, Oil_t is a binary variable which is unity when t denotes year 2006 and zero otherwise. We also consider medium-run effects of the shock by including lagged dummies Oil_{t-m} which equal one when the oil shock happens m years ago. In the baseline case, we set the maximum number of lags as three years for migration and consumption, and as eleven years for trade to get sufficient observations under its five-year data frequency. The key variable of interest to verify the importance of geography for economic linkages is $\sum_{n=0}^{T} \alpha_{4n}$, the linear combination of coefficient estimates for the interaction terms of the oil shock and bilateral distance.

Table 3 reports the regression results. Based on the coefficient estimates for the interaction terms, bilateral economic linkages exhibit strong spatial patterns. As is shown in columns (1) and (2), a 1% increase in bilateral geographic distance lowers migration and trade flows from another state to ND by 0.394% and 0.578% respectively due to the oil shock. This finding points to the barriers in these two channels that covary with geography which limit the scope of positive influences brought forth by ND's economic success. Consequently, residents from distant states are constrained from physically moving to or exporting goods to the booming state. Such barriers can also account for the spatial pattern of consumption. As is reported in columns (3) and (4), ND's per-capita consumption growth is larger in magnitude relative to that of geographically distant states. From column (3), a 1% increase in distance raises ND's relative consumption boost driven by its oil shock by 0.049%. For example, the cumulative consumption growth three years after the shock in Nebraska is 8.7% higher than in Florida. This result, which suggests that ND's consumption is more synchronized with neighboring states', indicates that geography plays an essential role in shaping the variation in consumption comovement. The result remains robust in column (4) where we adjust consumption for output differentials, which further implies that the degree of consumption risk sharing decreases in distance across economies, consistent with the empirical regularity we documented earlier.

To conclude the empirical section, we establish a gravity model of risk sharing and conduct an event study to verify that geographic distance plays an essential role in shaping consumption synchronization across states. These findings point to the existence of frictions in the channels of risk sharing that covary with distance. In the next section, we develop a theoretical model in which we examine the interplay among the channels and quantify their impacts on the level and comovement of consumption across regions.

3 Theoretical Model

3.1 Setup

In this section we develop a theoretical framework to examine the channels of consumption risk sharing across regions. The economy is populated by a continuum of infinitely-lived homogeneous households who reside in different regions indexed $i \in [1, \mathcal{I}]$. Regions are interconnected through trade, migration, and finance channels.

Each region produces two intermediate goods: tradables (T) and nontradables (NT). The production of intermediate goods in sector $s \in \{T, NT\}$ combines capital $K_{is,t}$ and labor $L_{is,t}$ with a Cobb-Douglas technology:

$$Y_{is,t} = A_{i,t} K_{is,t}^{\alpha} L_{is,t}^{1-\alpha}. \tag{6}$$

The region-specific productivity $A_{i,t}$ follows a joint AR(1) process for its vector $A_t \equiv$

$(A_{1,t}, A_{2,t}, ..., A_{\mathcal{I},t})$ subject to shocks $\epsilon_t \equiv (\epsilon_{1,t}, \epsilon_{2,t}, ..., \epsilon_{\mathcal{I},t})$:

$$A_t = \rho A_{t-1} + \epsilon_t, \tag{7}$$

where ρ is a persistence coefficient matrix for lagged productivity of all the regions. The contemporaneous correlations among regional shocks $\epsilon_{i,t}$ are captured by a covariance matrix denoted as Σ.

The final goods for consumption consist of tradables $C_{iT,t}$ and nontradables $C_{iNT,t}$:

$$C_{i,t} = C_{iT,t}^{\nu} C_{iNT,t}^{1-\nu}, \tag{8}$$

where ν captures the weight of tradables. The final goods for investment, whose price and quantity are denoted as $I_{i,t}$ and $P_{Ii,t}$, are specified as

$$I_{i,t} = I_{iT,t}^{\nu_I} I_{iNT,t}^{1-\nu_I}, \tag{9}$$

where investment adds to the capital stock in region i subject to depreciation δ

$$K_{i,t} = (1-\delta)K_{i,t-1} + I_{i,t}. \tag{10}$$

The market clearing conditions for factors of production and for nontradable goods in region i are respectively given by

$$K_{i,t} = K_{iT,t} + K_{iNT,t}, \qquad L_{i,t} = L_{iT,t} + L_{iNT,t}, \tag{11}$$

$$Y_{iNT,t} = C_{iNT,t} + I_{iNT,t}. \tag{12}$$

Meanwhile, tradable goods for consumption and investment in region i will be a CES bundle of intermediate tradable goods sourced from all the regions:

$$X_{iT,t} = C_{iT,t} + I_{iT,t}, \qquad \text{where} \qquad X_{iT,t} = (\sum_{j=1}^{\mathcal{I}} X_{ji,t}^{\frac{\phi-1}{\phi}})^{\frac{\phi}{\phi-1}}. \tag{13}$$

However, trade from j to i is subject to an iceberg cost $\tau_{ji} \geq 1$. Therefore, the aggregate price of tradables in region i is determined by the trade cost, as well as the price of j's output (denoted as $p_{j,t}$) summed across regions of origin:

$$P_{iT,t} = [\sum_{j=1}^{\mathcal{I}} (\tau_{ji} p_{j,t})^{1-\phi}]^{\frac{1}{1-\phi}}. \tag{14}$$

Bilateral trade flows from j to i at t will therefore be given by

$$X_{ji,t} = \pi_{ji,t} X_{iT,t}, \quad \textit{where} \quad \pi_{ji,t} = (\frac{\tau_{ji} p_{j,t}}{P_{iT,t}})^{-\phi}. \tag{15}$$

Besides, trade costs also enter the tradable goods' market clearing condition

$$Y_{iT,t} = \sum_{j}^{\mathcal{I}} \tau_{ij} X_{ij,t}. \tag{16}$$

In addition to trade, regions are integrated in the finance channel by holding each other's assets, whose dividend payout is calculated as capital income net of investment expenditure. Let $Y_{i,t} = Y_{iT,t} + Y_{iNT,t}$ be the aggregate output in region i and α be the capital share in production, region i's dividend is given by

$$D_{i,t} = \alpha p_{i,t} Y_{i,t} - P_{Ii,t} I_{i,t}. \tag{17}$$

The returns to the assets from region i include these dividends and the changes in asset prices denoted as $q_{i,t}$:

$$R_{i,t} = \frac{q_{i,t} + D_{i,t}}{q_{i,t-1}}. \tag{18}$$

In every region there is a mutual fund on behalf of its households that constructs a portfolio of assets from different regions. A household has the right to an equal share of the fund as long as it resides there. To simplify the portfolio choice problem, we assume households are myopic and expect themselves to stay in the region when deciding on investment for the next period.[8] Meanwhile, they incur costs when collecting financial gains earned from other regions. In particular, the literature on the gravity model of financial flows across countries, led by Portes and Rey (2005) and Okawa and Van Wincoop (2012), suggests that bilateral financial frictions covary with geographic distance.

[8]Under this assumption, households only care about the expected consumption per-capita in their region of residence during the next period, which appears in the Euler equation that determines their portfolios. A future extension of this baseline scenario is to relax the assumption and allow households to consider migration probabilities which prompt them to reduce saving and raise current consumption when making investment decisions.

In this spirit, we introduce bilateral financial friction $e^{-f_{ij}}$ as an iceberg trade cost region j incurs when repatriating financial gains from region i. The cost can be regarded as an asset transaction cost or tax, similar to the friction modeled in Heathcote and Perri (2004) and Tille and Van Wincoop (2010).[9] Moreover, we assume it is second order in magnitude (i.e. proportional to shocks in the model). This assumption allows us to use the perturbation method developed by Devereux and Sutherland (2011) to solve the portfolio choice problem. The method combines a second-order approximation of the Euler equations and a first-order approximation of other equations in the model. Specifically, region i's Euler equation follows

$$E_t[\frac{U'(c_{i,t+1})}{P_{i,t+1}}R_{i,t+1}] = E_t[\frac{U'(c_{i,t+1})}{P_{i,t+1}}e^{-f_{ji}}R_{j,t+1}], \qquad \forall j \in [1, \mathcal{I}]. \tag{19}$$

where $c_{i,t}$ denotes consumption per-capita and $P_{i,t}$ denotes the price level in region i.

We use the Euler equation to derive the solution to the portfolio choice problem. First, we assume assets from region $\mathcal{I}$ to be a numeraire asset and denote i's holding of j's assets as $\alpha_{j,i,t}$. Region i's external wealth position is therefore given by

$$\mathcal{W}_{i,t+1} = R_{\mathcal{I},t}\mathcal{W}_{i,t} + \sum_{j}^{\mathcal{I}} \alpha_{j,i,t}(e^{-f_{ji}}R_{j,t} - e^{-f_{\mathcal{I}i}}R_{\mathcal{I},t}) + p_{i,t}Y_{i,t} + T_{i,t} - P_{i,t}C_{i,t} - P_{Ii,t}I_{i,t}, \tag{20}$$

where $T_{i,t}$ denotes the tax transfer region i receives, which is introduced to capture fiscal policies that also play an essential role in risk sharing within a country.

The vector of excess returns to the other assets is introduced as R_x:

$$\hat{R}'_{x,t} = [\hat{R}_{1,t} - \hat{R}_{\mathcal{I},t}, \hat{R}_{2,t} - \hat{R}_{\mathcal{I},t}, ..., \hat{R}_{\mathcal{I}-1,t} - \hat{R}_{\mathcal{I},t}], \tag{21}$$

where $\hat{y}_t$ represents the log-deviation of any variable y from its steady state at t. Next, we evaluate the second-order Taylor expansion of the Euler equation 19 as

$$E_t[\hat{R}_{x,t+1} + \frac{1}{2}\hat{R}^2_{x,t+1} - (\sigma\hat{c}_{i,t+1} + \hat{P}_{i,t+1})\hat{R}_{x,t+1}] = -\frac{1}{2}\mathcal{F}_i + \mathcal{O}(\epsilon^3), \tag{22}$$

[9] Iceberg transaction costs are not the only way to introduce frictions in the financial channel. In particular, Okawa and Van Wincoop (2012) discuss alternative bilateral financial frictions, including information costs, which can also rationalize the spatial patterns of financial flows observed in both international and domestic contexts (see Portes and Rey (2005) and Coval and Moskowitz (1999)).

where $\hat{R}^{2'}_{x,t+1}$ denotes the vector of excess squared returns

$$\hat{R}^{2'}_{x,t+1} = [\hat{R}^2_{1,t+1} - \hat{R}^2_{\mathcal{I},t+1}, \hat{R}^2_{2,t+1} - \hat{R}^2_{\mathcal{I},t+1}, ..., \hat{R}^2_{\mathcal{I}-1,t+1} - \hat{R}^2_{\mathcal{I},t+1}], \tag{23}$$

and $\mathcal{F}_i$ denotes i's vector of financial frictions defined as

$$\mathcal{F}_i' = [f_{\mathcal{I}i} - f_{1i}, f_{\mathcal{I}i} - f_{2i}, ..., f_{\mathcal{I}i} - f_{\mathcal{I}-1i}], \tag{24}$$

whose k^{th} element represents the additional financial friction region i incurs when holding $\mathcal{I}$'s asset relative to k's. $\mathcal{O}(\epsilon^3)$ in equation 22 captures all terms of order higher than two.

Next we take the difference of any pair of regions' expanded Euler equations (22)

$$E_t[\sigma(\hat{c}_{i,t+1} - \hat{c}_{j,t+1}) + (\hat{P}_{i,t+1} - \hat{P}_{j,t+1})]\hat{R}_{x,t+1} = \frac{1}{2}(\mathcal{F}_i - \mathcal{F}_j). \tag{25}$$

The term in the bracket represents the inflation-adjusted consumption differential across regions. We denote it in the vector term for all the region-pairs under examination as $\hat{c}p$. Equation 25 can therefore be written as

$$E_t(\hat{c}p_{t+1}\hat{R}'_{x,t+1}) = \frac{1}{2}\mathcal{F} + \mathcal{O}(\epsilon^3), \tag{26}$$

where $\mathcal{F}$ stacks $\mathcal{F}_i' - \mathcal{F}_j'$ vertically in a $\frac{\mathcal{I}(\mathcal{I}-1)}{2} \times (\mathcal{I}-1)$ matrix for the $\frac{\mathcal{I}(\mathcal{I}-1)}{2}$ region-pairs being analyzed. Appendix C outlines the technical details of how we solve the portfolio choice problem in an example with three regions by evaluating the portfolio determination condition (equation 26). It follows from this equation that bilateral financial frictions in $\mathcal{F}$ affect cross-region consumption comovement $\hat{c}p$ through asset allocations.

Households' objective is to maximize their expected lifetime utility. At the beginning of every period, a household living in region i supplies labor, collects wage and financial income, and decides on consumption and investment. It derives utility from consumption $c_{i,t} = \frac{C_{i,t}}{N_{i,t}}$ and disutility from labor hours $l_{i,t} = \frac{L_{i,t}}{N_{i,t}}$ in its region of residence:

$$U_{i,t} = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \kappa\frac{l_{i,t}^{1+\eta}}{1+\eta}, \tag{27}$$

where σ captures the degree of risk aversion and $\frac{1}{\eta}$ is the elasticity of labor supply.

After earning and spending its income in region i, the household decides whether and where it wants to migrate. When it makes the decision, it takes into account a nonpecuniary migration cost $d_{ij} > 0$ when moving from region i to j. If it stays, the cost is set to zero ($d_{ii} = 0$). The household collects an idiosyncratic benefit $\omega_i \sim F(\omega)$ from being located in region i at the end of the period. ω_i can be considered as a non-monetary benefit, such as weather and culture, that adds to the utility of living in i. Following Artuc et al. (2010), we assume ω_i is i.i.d across households, time, and space. It is drawn from an extreme-value distribution with zero mean:

$$F(\omega) = \exp[-e^{\omega/\theta-\gamma}]. \tag{28}$$

Therefore, a household's expected value of being in region i at time t is

$$V_{i,t} = U_{i,t} + \beta E(V_{i,t+1}) + \sum_{j}^{\mathcal{I}} \int (\bar{\omega}_{ij,t} + \omega_{jt}) f(\omega_j) \Pi_{k \neq j} F(\bar{\omega}_{ij,t} - \bar{\omega}_{ik,t} + \omega_{jt}) d\omega_j. \tag{29}$$

From the three components on the right side of the equation, the expected value consists of the current utility the household obtains, the base value of staying in the region, and option value of moving from the region to others in the future. $\bar{\omega}_{ij,t}$ denotes the cutoff benefit that makes the household indifferent between staying in i and moving to j at t:

$$\bar{\omega}_{ij,t} \equiv \beta[E(V_{j,t+1}) - E(V_{i,t+1})] - d_{ij}. \tag{30}$$

Under the distributional assumption of ω, the share of migrants from i to j is

$$m_{ij,t} = \frac{\exp(\bar{\omega}_{ij,t}/\theta)}{\sum_{k=1}^{\mathcal{I}} \exp(\bar{\omega}_{ik,t}/\theta)}, \tag{31}$$

The law of motion for population in region i (denoted as N_i) given $m_{ij,t}$ hence follows

$$N_{i,t} = \sum_{j=1}^{\mathcal{I}} m_{ji,t-1} N_{j,t-1}. \tag{32}$$

With all the ingredients introduced, we proceed to characterize optimal consumption risk sharing across regions as a benchmark. Suppose there is a benevolent social planner whose objective is to maximize the sum of all the households' expected lifetime utility:

$$\max \sum_{t=0}^{\infty} \sum_{i}^{\mathcal{I}} \beta^t N_{i,t} \lambda_{i,t} \left(\frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \kappa \frac{l_{i,t}^{1+\eta}}{1-\eta}\right) \tag{33}$$

subject to the resource constraint

$$\sum_{i}^{\mathcal{I}}(N_{i,t}P_{i,t}c_{i,t}+P_{Ii,t}I_{i,t})=\sum_{i}^{\mathcal{I}}p_{i,t}Y_{i,t}+\sum_{i}^{\mathcal{I}}T_{i,t}. \tag{34}$$

$\lambda_{i,t}$ is the per capita weight that the social planner assigns to the utility of residents in region i at time t. The social planner's optimal decision rule for a pair of regions i and j should satisfy

$$\frac{\lambda_{i,t}}{\lambda_{j,t}}(\frac{c_{i,t}}{c_{j,t}})^{-\sigma}=\frac{P_{i,t}}{P_{j,t}}. \tag{35}$$

When asset markets are complete, the optimal consumption allocation in the competitive equilibrium coincides with the decision of the planner who assigns time-invariant weights to all the regions regardless of the realization of regional productivity shocks. Therefore, the ratio of $\lambda_{i,t}$ to $\lambda_{j,t}$ denoted as

$$\Lambda_{ij,t}=\frac{\lambda_{i,t}}{\lambda_{j,t}} \tag{36}$$

should be constant. Based on this analysis, the volatility of $\Lambda_{ij,t}$ over time reflects bilateral financial frictions because it captures the departure of consumption from the allocation under complete markets. As is argued by Fitzgerald (2012), $\Lambda_{ij,t}$ offers great flexibility since it does not depend on the assumption about the asset market structure or about the specific form the financial friction takes. However, it is easier to use the asset transaction cost f_{ij} we introduced earlier as a measure to quantify the magnitude of financial frictions for cross-region comparison and counterfactual exercise. Therefore, we will use $\Lambda_{ij,t}$ in the qualitative analysis and f_{ij} in the quantitative analysis in section 3.2.

3.2 Two-region Analysis

After describing the general setup including $\mathcal{I}$ regions, we analyze a two-region case to explain the mechanism through which different channels affect consumption risk sharing and illustrate how the channels interact with each other.

Before showing the quantitative results from numerical exercises, we conduct qualitative analysis to elucidate the intuition of how consumption risk sharing is achieved in a two-region framework. To keep this qualitative analysis tractable, we impose several simplifying assumptions temporarily: The two regions under examination, indexed 1 and 2, are perfectly symmetric. There is no endogenous labor supply, tax transfer, or capital accumulation. All goods are tradable subject to bilateral trade costs $\tau_{12}=\tau_{21}=\tau>1$.

Under these assumptions we analyze the cross-region ratio of any variable $x \equiv \frac{x_1}{x_2}$ whose deviation from the steady state is denoted as $\hat{x} = \log \frac{x - \bar{x}}{\bar{x}}$. Log-linearizing the goods market clearing condition (equations 15 and 16) and the social planner's allocation rule (equations 35 and 36) yields

$$\begin{aligned} \hat{Y} &= \Omega \hat{c} + \phi(\Omega^2 - 1)\hat{p} + \Omega \hat{L} \\ &= [\frac{\phi\sigma}{\Omega}(1 - \Omega^2) + \Omega]\hat{c} + \frac{\phi}{\Omega}(\Omega^2 - 1)\hat{\lambda} + \Omega \hat{L}, \end{aligned} \tag{37}$$

where $\Omega = \frac{1-\tau^{1-\phi}}{1+\tau^{1-\phi}}$. Based on equation 37, the response of relative per-capita consumption $\hat{c}$ to relative output $\hat{Y}$ driven by productivity changes, varies with trade costs τ through the coefficient Ω. When domestic and foreign goods are sufficiently substitutable ($\phi > 1$), higher trade costs impede consumption risk sharing because the relative consumption increases with relative output fluctuations:

$$\frac{\partial(\partial c/\partial Y)}{\partial \tau} > 0. \tag{38}$$

Meanwhile, three channels, represented by the other terms on the right hand side of equation 37, help absorb the impact of productivity shocks on consumption. In particular, the direction for the dynamics of the variables follows

$$\frac{\partial p}{\partial Y} < 0, \qquad \frac{\partial \lambda}{\partial Y} < 0, \qquad \frac{\partial L}{\partial Y} > 0. \tag{39}$$

To explain the economic interpretation of how these channels counteract output shocks to insulate consumption, we analyze a scenario where there is a relative negative output shock to region 1 ($\hat{Y} \downarrow$). First, a terms-of-trade appreciation ($\hat{p} \uparrow$) alleviates the shortfall of region 1's income and hence leaves its consumption less affected. Second, more financial resources, represented by $\hat{\lambda} \uparrow$, mitigates region 1's consumption fluctuation. Since λ can be interpreted as the relative Pareto weight, the rise of λ's value represents the situation where the utility of the residents in region 1 which inflicts the output loss is more valued by the social planner when allocating financial resources. Given this financial allocation, region 1's relative consumption does not decline as significantly. Third, migration of population out of region 1 ($\hat{L} \downarrow$) reduces the local population among which resources are allocated and therefore equalizes consumption per-capita across regions.

We now proceed to conduct numerical exercises and analyze the quantitative results of the model. The framework is similar in style to the workhorse model in international macroeconomics developed by Backus et al. (1992) who examine the real business cycles of two symmetric economies. We enrich the framework by incorporating trade, migration, and asset flows under frictions across economies. In terms of parameterization, the model is calibrated to the U.S. annual data for cross-state analysis. Table 4 summarizes the parametric assumptions under which the baseline two-region framework is solved. First, we adopt the standard assumptions from macroeconomic literature (listed in panel (I)) including the coefficient of risk aversion and elasticity of labor supply.

In panel (II), we report the parameters estimated from the U.S. aggregate economy. Specifically, we estimate labor share in production $1-\alpha$ to be 0.59 by dividing the labor earnings by the output data, both from the BEA, over the period of 1977-2019. In addition, we set the share of consumption expenditure on tradables (ν) as 0.31 following Johnson (2017), who estimates the value based on the US CPI expenditure data from the BEA. Moreover, we set the weight of tradables in investment (ν_I) as 0.4 following Bems (2008). His analysis uses the input-output table from the OECD. Last but not least, we follow Simonovska and Waugh (2014) and Artuc et al. (2010) when setting elasticities of trade and migration respectively.

In panel (III), we characterize the joint productivity process of a pair of states. We choose Georgia and Ohio (GA and OH for brevity), the median states in terms of output per capita, as our sample of analysis. We first calculate the total factor productivity (TFP) proxied by the Solow residual in each region $i \in \{\text{GA, OH}\}$ at time t from

$$\log(A_{i,t}) = \log(Y_{i,t}) - \alpha \log(K_{i,t}) - (1-\alpha)\log(L_{i,t}), \tag{40}$$

where $Y_{i,t}$ and $L_{i,t}$ are output and number of employees in state i in year t from the BEA. State-level capital stock $K_{i,t}$ is not directly available, so we construct the measure following Garofalo and Yamarik (2002)'s method. Specifically, we apportion national capital stock to states based on their industry-level income data (see Appendix B for details). After we calculate the state-level TFP, we detrend the series with the HP filter and estimate a joint AR(1) process (specified in equation 7) assuming the shocks are serially independent and drawn from a joint normal distribution. Table 4 reports the persistence and covariance matrices of Georgia and Ohio's productivity.

Panel (IV) of table 4 lists the values of bilateral frictions calibrated to the pair of states under examination. Trade costs, migration costs, and financial costs are estimated to match three targeted moments: the mean export-to-output ratio (0.392), the mean emigrant-to-population ratio (0.028), and the bilateral consumption correlation (0.824) of Georgia and Ohio over the sample period. When estimating trade and migration frictions simultaneously, we start with an initial guess for the combination of the two frictions, and solve for the corresponding wage rates and labor hours given the frictions that satisfy the labor market clearing condition. Then we update the guess and repeat the procedure until the model-predicted export-to-output and emigrant-to-population ratios converge to those in the data. After calibrating these two frictions on the real side of the economy, we estimate model-consistent financial frictions. Unlike trade and migration whose bilateral flows are available in the data, state-to-state financial flows are not directly observable. Therefore, we infer financial frictions indirectly from the consumption pattern. Besides its feasibility given limited data availability, this method is helpful in capturing the influences of the financial channel on consumption comovement. Calibrating financial frictions with this method involves three steps. First, we obtain the coefficient matrices necessary to solve the portfolio choice problem from the first-order conditions of the model.[10] Second, we solve for asset holdings $\tilde{\alpha}$ under which the model-implied bilateral consumption correlation exactly matches that in the data. Third, we plug the calibrated portfolio in equation 26 to recover financial frictions.

Table 4: Parametrization

Parameter	Description	Value	Source
	(I)		
β	Annual discount factor	0.95	
σ	Coefficient of relative risk aversion	1	Macroeconomic
δ	Capital depreciation	0.06	Literature
η	Inverse of elasticity of labor supply	0.5	
	(II)		
ν	Weight of tradables in consumption	0.31	Johnson (2017)
ν_I	Weight of tradables in investment	0.40	Bems (2008)
α	Capital intensity in production	0.41	BEA
θ-1	Elasticity of trade	4.1	Simonovska and Waugh (2014)
ϕ	Elasticity of migration	4.5	Artuc et al. (2010)
	(III)		
ρ	Persistence matrix of productivity	$\begin{bmatrix} 0.65 & 0.06 \\ 0.04 & 0.53 \end{bmatrix}$	Estimated from GA and OH's TFP
Σ	Covariance matrix of shocks	$\begin{bmatrix} 1.21 & 1.25 \\ 1.25 & 2.56 \end{bmatrix} e^{-4}$	
	(IV)		
τ	Trade cost	1.031	Calibrated to match GA and OH's mean
d	Migration cost	19.58	export-to-output, emigrant-to-population,
f	Financial cost	3e-5	and consumption comovement

Under the specified parametrization, table 5 compares the contemporaneous correlations of variables in the calibrated model with those in the data. Panel (I) reports

[10]Appendix C provides the technical details. The coefficient matrices include R_1, R_2, D_1, and D_2 in equations A17 and A18, which capture the responses of consumption differentials and excess asset returns to excess portfolio returns.

Table 5: Contemporaneous Correlations of Variables

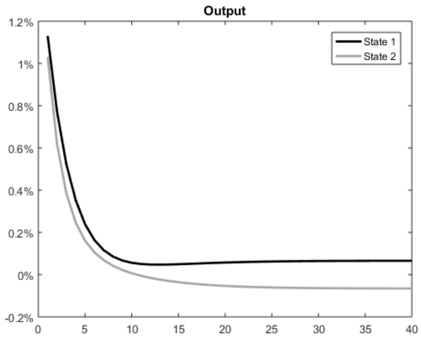
	Model	Data
	(I) Cross-state Correlation	
Output $\rho(Y_1, Y_2)$	0.85	0.84
Consumption $\rho(C_1, C_2)$	0.79	0.78
Output per capita $\rho(y_1, y_2)$	0.84	0.88
Consumption per capita $\rho(c_1, c_2)$	0.82	0.82
	(II) Correlation with Self Output	
Consumption per capita $\rho(c, y)$	0.95	0.91
Net exports $\rho(NX/Y, Y)$	-0.04	-0.03
Population $\rho(N, Y)$	-0.01	-0.02

This table reports the contemporaneous correlations of HP filtered data and those in the calibrated model. Panel (I) reports the cross-region comovement of output and consumption at the aggregate (denoted as Y_i, C_i) and per capita (denoted as y_i, c_i) levels. Panel (II) reports the comovement of a region's scaled net exports (NX/Y) and population (N) with its own output, as well as the correlation between its consumption and output per capita.

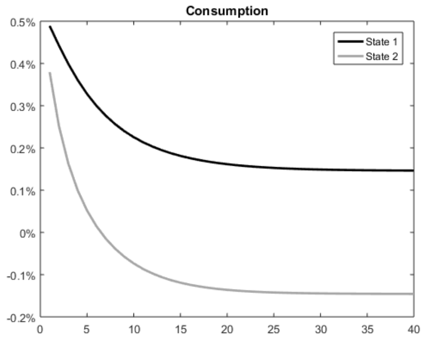
the cross-state comovement of output and consumption. The model performs well in matching empirical moments at both the aggregate and the per capita levels. In either case, output exhibits stronger cross-state synchronization than consumption. This result, which verifies the consumption correlation puzzle in the empirical section, points to the existence of frictions that impair risk sharing. Panel (II) presents the correlation between a state's consumption with its own output per capita. Based on the finding that the correlation is greater than 0.9 in both the model and the data, consumption is highly procyclical. Furthermore, Panel (II) reports the correlation between a state's scaled net export (NX/Y) and population (N) with its own output (Y). Scaled net exports, measured as the ratio of the difference between exports and imports to output, turn out to be countercyclical. This finding is consistent with the international stylized facts documented by Mendoza (1991) and Backus et al. (1992). In addition, the contemporaneous correlation between population and output is negative both empirically and theoretically. Nevertheless, this correlation does not reflect the cumulative effects caused by delayed migration decisions under migration costs. To overcome such limitations, we examine the dynamic response of variables by plotting impulse response functions.

Figure 2 shows the impulse responses of the two states' output and consumption to a one-standard-deviation innovation in state 1's productivity. The black line shows the dynamics of state 1's variables and the grey line shows state 2's. Both states experience increases in output and consumption right after the productivity shock takes place. Even though the shock happens to state 1, there is positive spillover to state 2 not only due to productivity covariances but also thanks to cross-state goods, financial, and labor flows.

Figure 2: Cross-state Comparison of Impulse Response Functions



(a) Output



(b) Consumption

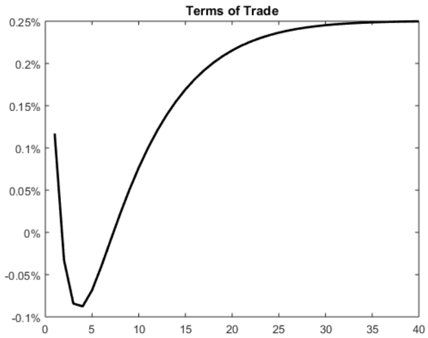
Note: This figure plots the dynamic responses of macroeconomic variables, including output and consumption, to a one-standard-deviation innovation in state 1's productivity. The black and grey lines respectively show state 1's and state 2's variables. These variables are measured as a percentage of steady-state output in the plots.

Nevertheless, synchronization across states is not perfect and therefore state 1 witnesses greater improvements in its output and consumption.

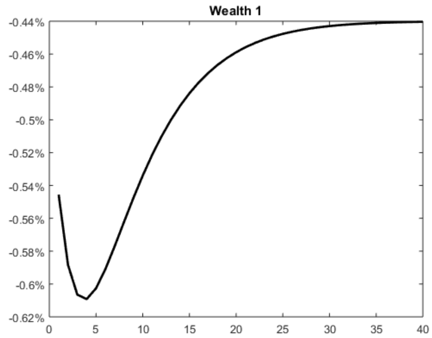
To further understand the driving forces of synchronization, we examine the key variables of interest in the three channels. Figure 3 plots the impulse responses of state 1's terms of trade, external wealth position, investment, and population. Following a positive productivity shock to state 1, state 1 experiences a terms-of-trade depreciation as its exports become cheaper relative to imports to clear the goods market. This depreciation will help increase the consumption of state 2, which does not experience the productivity boost, by raising its relative nominal income. Meanwhile, state 1 has a negative external wealth position, which suggests that it borrows from state 2. This could be understood from the fact that capital resources are allocated to the more productive economy where returns to investment are higher, which causes state 1's investment spike shown in figure 3c. As is argued by Heathcote and Perri (2013), this cross-border investment financing facilitates risk sharing. Lastly, population flows into state 1 (figure 3d), which raises the number of households among whom the increased aggregate consumption is shared and therefore helps to equalize consumption per capita across states. These quantitative results are mostly consistent with the qualitative analysis based on equation 37, with the exception that endogenous capital accumulation, which is absent from the qualitative analysis, alters the direction of financial flows in the short run, the same prediction as the one from the international RBC framework by Backus et al. (1992).

In the next step we conduct comparative analyses by varying the magnitude of the

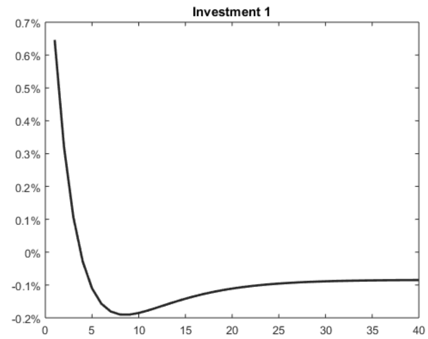
Figure 3: Impulse Response of State 1's Macroeconomic Variables



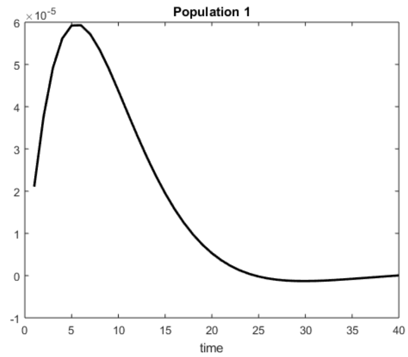
(a) Terms of Trade



(b) External Wealth



(c) Investment

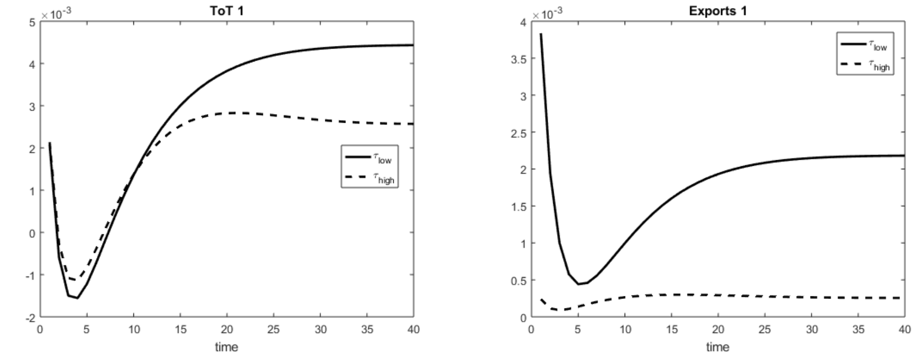


(d) Population

Note: This figure plots the dynamic responses of macroeconomic variables to a one-standard-deviation innovation in state 1's productivity. Variables under examination include state 1's terms of trade, external wealth, investment, and population. They are measured as a percentage of steady-state output in the plots.

frictions to understand the impacts of barriers on the effectiveness of as well as the interactions among the channels of consumption risk sharing. Figures 4-6 illustrate the scenarios in which one type of friction doubles its calibrated value while the other parameters remain unchanged as in the baseline case. In the trade channel, state 1's terms-of-trade and exports to state 2 are less volatile when trade costs are high, as is shown in figure 4. This finding suggests that higher trade costs mute trade adjustments to productivity innovations, which leaves state 2 less benefitted from state 1's positive productivity shock. In the financial channel (figure 5), financial frictions raise state 1's cost of holding state 2's assets and generate asset home bias. However, the dividends to assets, calculated as the difference between capital income and investment expenditure, are lower for state 1's assets than for state 2's given state 1's investment spike driven by the productivity shock. Therefore, in figure 5a higher financial frictions lower state 1's wealth accumulation by tilting portfolios toward temporarily lower-yielding domestic assets. Meanwhile, the financial channel has spillover effects on the migration channel by altering households' migration decisions. Lower financial frictions facilitate consumption risk sharing by allowing states to hold each others' assets, which dampens households' incentive to physically move across states in pursuit of higher consumption. Therefore, the dynamics of population are less volatile when financial frictions are lower in figure 5b.

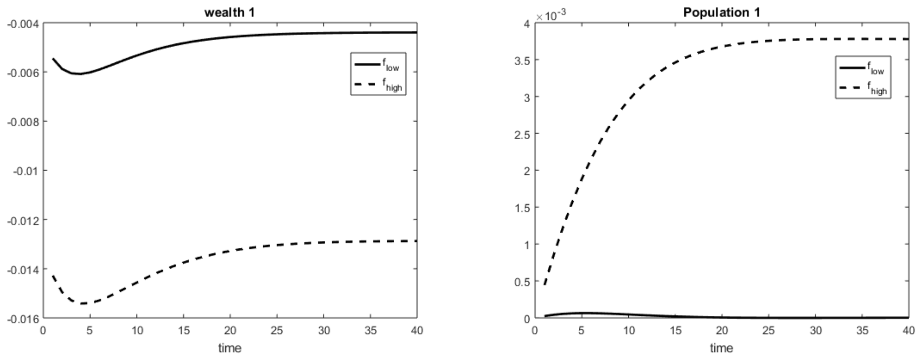
Figure 4: Comparative Analysis under Different Trade Costs



(a) Terms of Trade (b) Exports

Note: This figure plots the dynamic responses of state 1's terms of trade and exports to state 2, to a one-standard-deviation innovation in state 1's productivity. The solid and dashed lines show the situations with low and high trade costs respectively.

Figure 5: Comparative Analysis under Different Financial Frictions



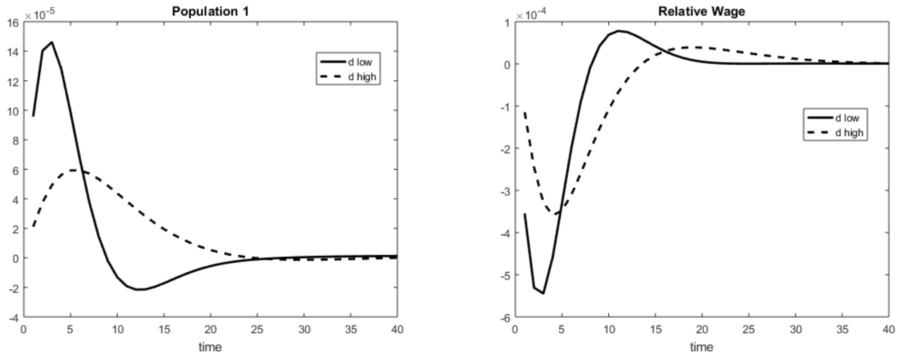
(a) Wealth (b) Population

Note: This figure plots the dynamic responses of state 1's external wealth and population, to a one-standard-deviation innovation in state 1's productivity. The solid and dashed lines show the situations with low and high financial frictions respectively.

The response of migration is also smaller when migration costs are higher, as is shown in figure 6a that raising migration costs flattens the curve of cross-state population flows.

Under higher migration costs, not only is the magnitude of migration smaller, but also the duration of population flows is longer before reaching the new steady state. The hump-shaped migration pattern is driven by the forward-looking migration decisions subject to migration frictions. Moreover, the dynamics of the relative wage rate across states denoted as $w = \frac{w_1}{w_2}$ is depicted in figure 6b, which appears almost as a mirror image of figure 6a under the labor market clearing condition. The figure shows that higher migration costs cause smoother fluctuations in the relative wage. For example, the plunge of the relative wage right after state 1's positive productivity shock is larger when migration costs are lower. To understand this result, w_1 falls more relative to w_2 due to the terms-of-trade depreciation that reduces state 1's nominal marginal product of labor. If migration were to take place that drew more population to state 1 in response to its higher consumption growth, w_1 would decline even further to clear the labor market. Therefore, higher migration costs avoid a greater plummet in the relative wage and therefore increase wage synchronization across states.

Figure 6: Comparative Analysis under Different Migration Costs

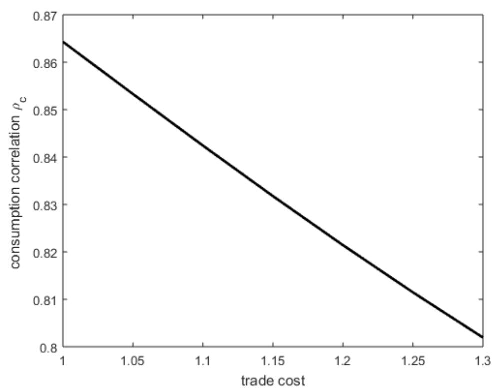


(a) Population (b) Relative wage

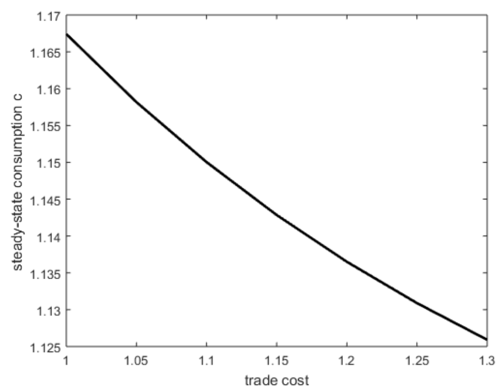
Note: This figure plots the dynamic responses of state 1's population and relative wage $w = \frac{w_1}{w_2}$, to a one-standard-deviation innovation in state 1's productivity. The solid and dashed lines show the situations with low and high migration costs respectively.

Based on these discussions, bilateral economic linkages through trade, migration, and finance affect risk sharing across regions. Frictions in these channels should therefore have important implications for cross-state consumption comovement. We conduct another set of comparative analyses to test this hypothesis. Specifically, we calculate the model-predicted consumption correlation when changing the counterfactual value of one friction at a time. This exercise involves three steps. Step 1, we calculate the equilibrium values of all the variables on the real side of the economy under specific trade and migration frictions. Step 2, we solve the portfolio choice problem under financial frictions by evaluating the first- and second-order dynamics of the model. Step 3, we simulate the model that encompasses both real and financial allocations of the economy and compute the resulting bilateral consumption comovement in these counterfactual scenarios.

Figure 7: Consumption under Different Trade Costs



(a) $\rho(c_1, c_2)$

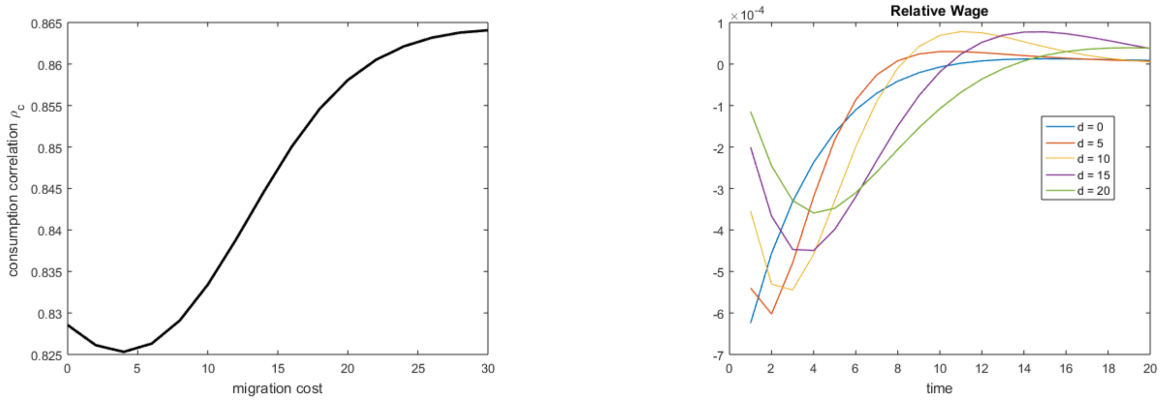


(b) $\bar{c}_i$

Note: This figure plots the pattern of consumption per capita under different trade costs. Figure 7a plots the correlation coefficient of consumption per capita across states given different trade costs. Figure 7b plots the state-level consumption per capita in the steady state of the economy.

Figure 7 shows the pattern of consumption per capita under different trade costs. Figure 7a, which plots the correlation coefficient of consumption per capita across states $\rho(c_1, c_2)$, suggests that higher trade costs hinder cross-state consumption comovement. For example, consumption correlation rises from 0.802 under calibrated trade costs to 0.864 when there is no trade cost ($\tau = 1$). Besides raising the correlation coefficient as a second-moment variable, lowering trade costs also raises the level of consumption, a first-moment variable. Figure 7b illustrates the state-level consumption per capita in the steady state of the economy. The level of consumption increases from 1.13 to 1.17 when the trade cost decreases from 1.3 to 1, which is caused by the smaller loss of tradable goods during transportation under lower iceberg trade costs. Based on these findings, eliminating trade costs raises consumption and facilitates cross-state risk sharing.

Figure 8: Consumption under Different Migration Costs



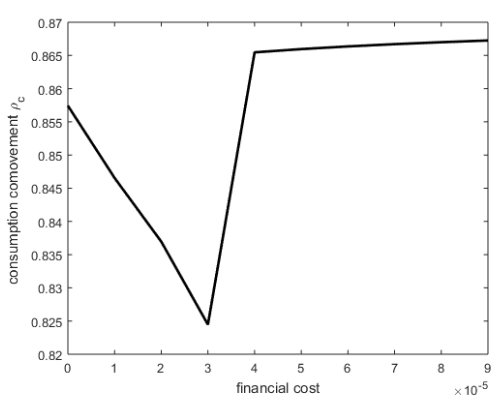
(a) $\rho(c_1, c_2)$

(b) Impulse response of w

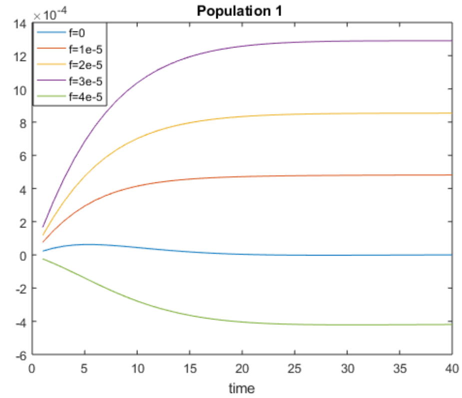
Note: Figure 8a plots the correlation coefficient of consumption per capita across states given different migration costs. Figure 8b plots the impulse response of the cross-state wage ratio $w = \frac{w_1}{w_2}$ to a one-standard-deviation innovation in state 1's productivity.

Figure 8 shows the pattern of consumption per capita under different migration costs. The cross-state consumption correlation shown in figure 8a does not change monotonically with migration costs as with trade costs. When the costs first emerge around the neighborhood of zero, consumption comovement decreases, which suggests that higher migration costs impair consumption risk sharing. After that, consumption correlation increases with migration costs, although the concave curve suggests diminishing marginal effects of the migration costs. The shape of the curve is largely driven by the impact of the migration costs on the relative wage across states (denoted as $w = \frac{w_1}{w_2}$), whose impulse responses are plotted in figure 8b. Consistent with the earlier analysis for figure 6b, higher migration costs reduce the dynamics of the wage ratio through flattening population flows over time. A smoother relative wage pattern suggests a greater correlation of wage rates across states, which leads to a higher degree of consumption comovement since labor income is an important funding source for households' consumption expenditure. This explains the reason that higher migration costs raise the correlation coefficient of consumption in general (figure 8a). The exception to this general pattern happens when migration costs are too low to generate a smooth cross-state wage convergence (shown as the kink of the red line in figure 8b). Under this circumstance, consumption comovement deteriorates under higher migration costs.

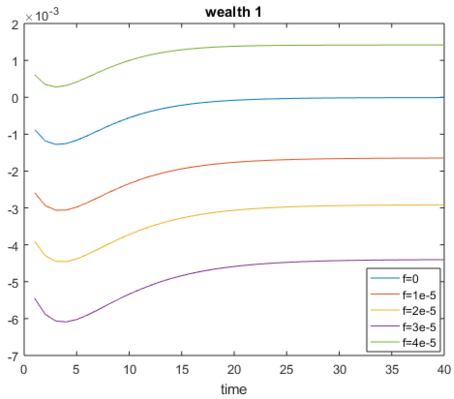
Figure 9: Consumption under Different Financial Frictions



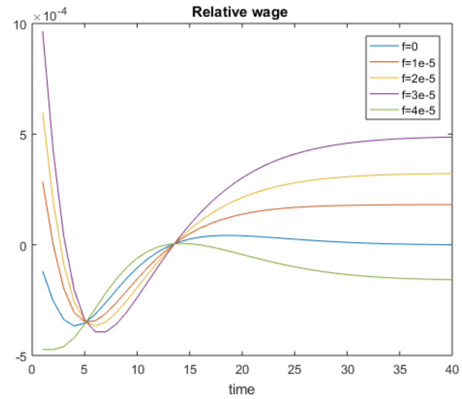
(a) $\rho(c_1, c_2)$



(b) population 1



(c) wealth 1



(d) relative w

Note: Figure 9a plots the correlation coefficient of consumption per capita across states given different financial frictions. Figures 9b-9d plot the impulse responses of state 1's population, wealth, and cross-state wage ratio $w = \frac{w_1}{w_2}$ to a one-standard-deviation innovation in state 1's productivity.

Figure 9 explores the patterns and determinants of consumption comovement under different financial frictions. As is shown in 9a, consumption correlation does not vary monotonically or smoothly with financial costs. Around the neighborhood of the calibrated financial friction $[0, 3]e^{-5}$, higher financial frictions lead to weaker consumption comovement. This is consistent with our analysis earlier: financial frictions raise the cost of holding foreign assets and tilt portfolios more toward domestic assets. Consequently, each state's consumption, driven more by its own output performance, is less synchronized with each other. What causes the discontinuity in figure 9a is the drastic change in the migration pattern shown in 9b. To understand this result, recall from the analysis for figure 5 that higher financial costs reduce state 1's wealth accumulation in response to its positive productivity shock (see figure 9c). When financial frictions are sufficiently large ($\geq 4e^{-5}$), the deterioration of state 1's wealth position starts to negatively affect its consumption and hence induces population to move from state 1 to 2 instead. Based on the same analysis as for figure 6b, this migration from state 1 to 2, by raising the relative wage of state 1, counteracts the impact of terms-of-trade depreciation after state 1's productivity shock. Therefore, the wage ratio across states is less volatile (figure 9d), which suggests that cross-state wage comovement is stronger. Given the importance of labor income for consumption expenditure, consumption correlation is stronger when financial costs are high enough to shift migration. Therefore, the three channels of consumption risk sharing are all manifested in figure 9a where they jointly shape the pattern of consumption correlation under financial frictions. This analysis underscores the importance of examining the interaction of these channels in the general equilibrium.

To conclude this section, we develop a theoretical model in which cross-region consumption synchronization is shaped by three channels: migration, trade, and finance. We use a two-region example calibrated to the US data to elucidate the interplay among the three channels and their joint effects on the consumption pattern. In the next section we extend the two-region case to a multi-region scenario to conduct a more comprehensive and realistic quantitative analysis of the theoretical model.

4 Quantitative Assessment

This section evaluates the theoretical model quantitatively in a multi-region DSGE framework, which allows us to quantify the magnitude and influence of various frictions on spatial consumption comovement. Furthermore, we conduct counterfactual analyses to deliver implications for macroeconomic policies.

4.1 Extended Model

We enrich the framework in section 3.2 by relaxing the symmetric two-region assumption. First, the equilibrium population size is different across states and taken from their values from the data. Second, we extend the two-region to a multi-region case so that multilateral economic exchanges clear the goods, labor, and financial markets in aggregate. This extension allows us to examine the total effects of bilateral economic linkages on each region.

Calibrated to the U.S. state-level data, the model encompasses $\mathcal{I} = 50$ regions. Ideally, a household in region i considers all the $\mathcal{I}$ regions when making economic decisions.

One computational challenge we face when solving the multi-region DSGE model is that the large matrix that covers the bilateral ties for all the regions is badly scaled given the uneven distribution of economic sizes. Therefore, using this matrix to derive portfolio choice with the perturbation method yields inaccurate results. To overcome this challenge, we propose a trilateral framework when analyzing any region pair. The framework consists of $i, j \in \mathcal{I}$, as well as the rest of the economy from i and j's perspective (ROE for simplicity). This trilateral framework not only enables the examination of bilateral frictions between i and j, but also considers the impacts of multilateral resistance of all the other regions affecting the region-pair. The latter echoes the extended gravity model in the trade literature developed by Anderson and Van Wincoop (2003) to capture the substitutability across trade partners.

In terms of calibration, many parameters take the same values from the existing literature as in the two-economy framework summarized in table 4. For state-specific parameters, we follow the same strategy as in section 3.2 with modifications tailored to the trilateral framework. For example, we follow the literature on risk sharing including Corsetti et al. (2008) to characterize productivity as the Solow residual. The variables of ROE, denoted with asterisks below, will be the sum of all the $\mathcal{I}$ regions' variables minus i and j's. Therefore, ROE's productivity at time t is computed from

$$\begin{aligned} \log(A_t^{ij*}) &= \log(Y_t^{ij*}) - \alpha \log(K_t^{ij*}) - (1-\alpha)\log(L_t^{ij*}) \\ &\equiv \log(\sum_i^{\mathcal{I}} Y_{i,t} - Y_{i,t} - Y_{j,t}) - \alpha \log(\sum_i^{\mathcal{I}} K_{i,t} - K_{i,t} - K_{j,t}) \\ &- (1-\alpha)\log(\sum_i^{\mathcal{I}} L_{i,t} - L_{i,t} - L_{j,t}). \end{aligned} \tag{41}$$

Next we obtain the variance-covariance matrix (Σ) of these three regions' productivity assuming the annual persistence of productivity is 0.72, which is estimated from the U.S. country-level Solow residual. We estimate Σ for all the $\frac{1}{2}\frac{\mathcal{I}}{\mathcal{I}-1} = 1225$ state pairs.

Another distinct feature of this asymmetric framework is that each region may not run a balanced budget in the equilibrium. To this end, we collect the data on state-level output and expenditure (defined as the sum of consumption and investment), whose difference represents the net asset position of the economy. ROE's asset position will be the sum of all the states' positions minus the positions of the state-pair under examination. The time-averaged asset positions will be reflected in the portfolio choice to solve for.

We now proceed to discuss the calibration strategies for bilateral frictions in the trilateral framework. There are three economies numbered $1, 2, 3$ with 1 and 2 representing the pair of states being studied and 3 representing ROE. The three economies encounter a set of six bilateral frictions in each of the trade, migration, and finance channels

$$\{x_{12}, x_{13}, x_{23}, x_{21}, x_{31}, x_{32}\}, \qquad x \in \{\tau, d, f\}. \tag{42}$$

In terms of trade and migration costs, we estimate them simultaneously to ensure that the model-predicted bilateral migration and trade linkages match those from the IRS and CFS data. The estimation procedure is similar to that in section 3.2: Step 1, we start with an initial guess for the combination of migration and trade costs. Step 2, we solve for wage rates and labor hours given the frictions that satisfy the labor market clearing condition. Step 3, we calculate the corresponding trade shares ($\pi_{ij,t}$ in equation 15) and migration shares ($m_{ij,t}$ in equation 31) to the wages solved earlier. Step 4, we repeat the previous steps until the trade and migration shares converge to the empirical moments.

After characterizing the real side of the model, we calibrate frictions in the financial channel to the pattern of consumption comovement across economies. Specifically, we estimate the coefficients of consumption risk sharing among the three economies with the same data and method as in the empirical section

$$\beta = [\beta_{12}, \beta_{13}, \beta_{23}], \tag{43}$$

and use the coefficients as targeted moments to estimate bilateral frictions. Appendix C outlines the technical details of the portfolio choice problem in this trilateral framework. The algorithm is slightly modified from that in section 3.2: First, we obtain the coefficient matrices, including R_1, R_2, D_1, D_2 in equations A17-A18 necessary to solve the portfolio choice problem from the first-order dynamics of the model. Second, we solve for asset holdings under which the model-implied risk-sharing coefficients β match those estimated from the data. To simplify our computation in this step, we assume a state's holding of ROE's assets is the same whose baseline weight in the portfolio is one-half but the state can choose the remaining composition between its own and pair partner's assets under risk-sharing motives. Third, we plug the calibrated asset positions in the portfolio determination equation (equation A13) to compute financial frictions.

Our benchmark calibration is based on the sample period from 1997 to 2017. The sample selection is largely driven by the availability of the CFS trade data. We use the time-averaged state-level population, net asset positions, trade and migration flows as

Table 6: Estimated Financial Frictions and Banking Linkage

Dep. Var: Est. Frictions $\log(\hat{f}_{ij})$	(1)	(2)
Branches	-5.7e-04***	
	(1.1e-04)	
Deposits		-6.8e-09***
		(1.6e-09)
Observations	2442	2442
R^2	0.001	0.001

Robust standard errors in parentheses, *** significant at 1%. The independent variable is the estimated bilateral financial friction between states i and j. Dependent variables include the number of bank branches, and the dollar amount of deposits collected by financial institutions, located in i and headquartered in j. Estimated frictions are missing for few pairs because the eigenvalues computed at the steady state of the model for those pairs do not satisfy the Blanchard and Kahn (1980) condition to guarantee the existence of a unique solution.

the steady-state values of those variables when estimating and solving the model. We evaluate the model fit by comparing empirical and model-predicted moments of bilateral variables. Figure A.2 presents the performance of the model in matching targeted moments including bilateral trade shares, bilateral migration shares, and coefficients of consumption risk sharing. From the figures, the model does a good job matching these empirical moments since most of the observations fall on the 45-degree line. In terms of untargeted moments, the key variable of interest is bilateral consumption correlation. To obtain its predicted value from the quantitative model, first we compute the steady-state values of all the endogenous variables in this calibrated DSGE model. Then we simulate the model with calibrated productivity shocks and examine the impulse responses of consumption per capita in different states. Lastly, we compute consumption correlation, averaged over the simulated shocks, and compare it to the empirical moments. Figure A.3 suggests that the model does modestly well in predicting bilateral consumption comovement. For example, the model successfully predicts that Wyoming exhibits a notably stronger consumption correlation with Texas and West Virginia than Massachusetts.

Among the three frictions, we are particularly interested in testing whether our estimated financial frictions are reasonable. To this end, we collect the Federal Deposit Insurance Corporation (FDIC) bank statistics, which list branch locations and their reported deposits. States i and j are deemed to exhibit stronger financial ties when banks headquartered in i open more local branches in j or collect more deposits from branches located in j. Therefore, we compile this information of all the FDIC-insured institutions and explore its consistency with financial frictions $\hat{f}_{ij}$. Based on the results presented in table 6, an increase of one thousand branches or one billion deposits collected by institutions, located in i and headquartered in j, is associated with a decrease of .57% or 6.8% estimated financial frictions ($\hat{f}_{ij}$) respectively. This analysis provides external validity for our estimates: Financial frictions estimated from the consumption data are consistent with empirical evidence from the banking sector.

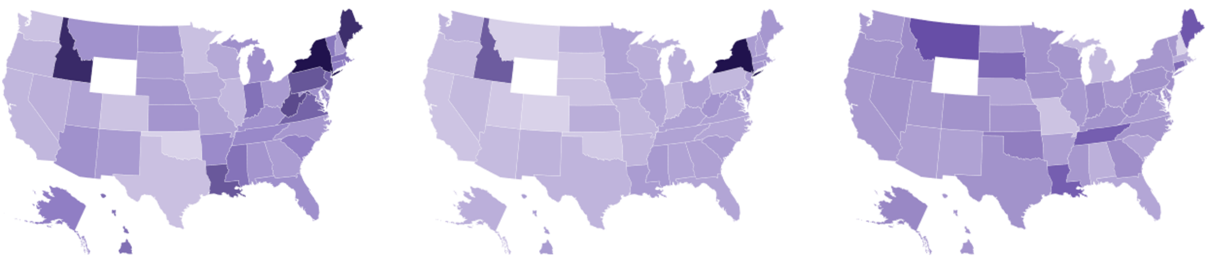
After evaluating the model performance, we now proceed to discuss numerical predictions from the quantitative model.

4.2 Numerical Results

This section presents the predictions of the quantitative model. First, we evaluate the magnitude of the estimated frictions. Second, we conduct counterfactual analyses to quantify the effects of each friction. Lastly, we solve for optimal macroeconomic policies which, by offsetting the impacts of frictions on consumption, improve social welfare.

To provide a first glance of the frictions in the three channels of risk sharing, we use Wyoming as an example by showing the heatmaps of its estimated bilateral frictions with other states in figure 10. Each type of bilateral friction is calculated as the geometric mean of outbound and inbound frictions ($x_{WY,i},\ x_{i,WY},\ i \in [1, \mathcal{I}], x \in \{\tau, d, f\}$) between Wyoming (in white) and any other state. In general, states located within a smaller radius from Wyoming exhibit lower frictions with the state. For example, the migration cost between Wyoming and a neighboring state Colorado is the lowest, whose value is approximately 1/3 of that between Wyoming and Hawaii. This spatial pattern is consistent with the observation in figure 1 that Wyoming shows stronger economic linkages with states in closer proximity. However, there are exceptions to the pattern. Idaho, another neighboring state of Wyoming, is estimated to inflict high trade costs due to its low trade volume with Wyoming unexplained by the size of its expenditure.

Figure 10: Wyoming's Estimated Frictions with Other States



(a) Trade (b) Migration (c) Finance

This figure plots the estimated bilateral frictions between Wyoming (in white) and other states in the U.S. A darker color suggests a higher value of friction. Frictions are calculated as the geometric average of bidirectional frictions (inbound friction to and outbound friction from Wyoming) in each of the channels.

Table 7: Bilateral Frictions and Geographic Distance

Dep. Var: Est. Frictions	$\log(\hat{\tau_{ij}})$	$\log(\hat{d}_{ij})$	$\log(\hat{f}_{ij})$
$\log(dist_{ij})$	0.525 ***	0.100 ***	0.232 **
	(0.047)	(0.01)	0.097
Observations	2442	2442	2442
R^2	0.041	0.023	0.003

This table reports the regression results of equation 44. Robust standard errors in parentheses, standardized coefficients in brackets. *** significant at 1%, ** significant at 5%. Estimated frictions are missing for few pairs because the eigenvalues computed at the steady state of the model for those pairs do not satisfy the Blanchard and Kahn (1980) condition to guarantee the existence of a unique solution.

To explore the geographic characteristics of the three frictions in general, we run bivariate regressions with the estimated bidirectional frictions as dependent variables and geographic distance as the independent variable for all the $\frac{\mathcal{I}(\mathcal{I}-1)}{2}$ state pairs:

$$\log(\hat{x}_{ij}) = \alpha_x + \gamma_x \log(dist_{ij}) + \epsilon_{ij}, \qquad x \in \{\tau, d, f\}. \tag{44}$$

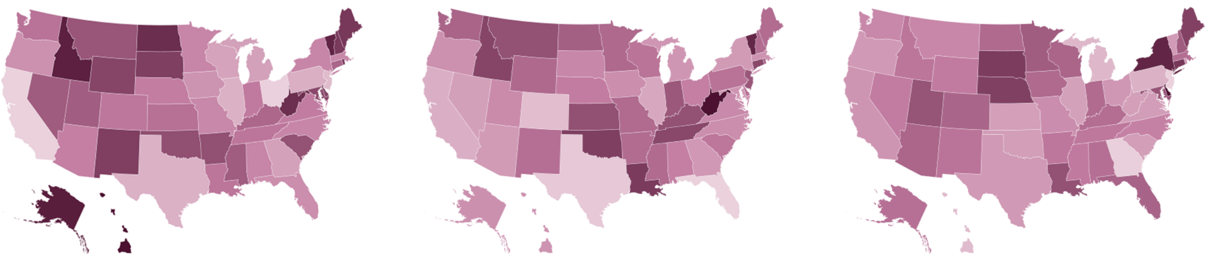
As reported in table 7, a 1% rise in distance is associated with a 0.525% increase in trade costs, a 0.100% increase in migration costs, and a 0.232% increase in financial frictions. By comparing these values, we infer that trade costs are most sensitive to distance. All the coefficient estimates ($\hat{\gamma_x}$ in regression 44) are significantly positive. This numerical result confirms one of the key hypotheses of this paper that frictions which impair risk sharing covary with geographic distance between states, which potentially shapes the spatial pattern of consumption.

We now evaluate and compare frictions at the state level. In particular, we compute the median frictions across all the state pairs each state forms and present them in figure 11 and table A.4. Based on the estimation results, the trade costs of Hawaii and Alaska, the two non-contiguous states, are among the highest. For example, the median outbound trade cost of Alaska is 3.14 times the cost of Georgia and Ohio, the median states in terms of output per capita. In the migration channel, Florida and Texas are estimated to face the lowest inbound migration costs. This finding coincides with the observation that these two states are popular destinations of migration inflows in recent decades. In the financial channel, states whose estimated financial frictions are the highest include Delaware, Alaska, and Nebraska. This result is largely driven by the relatively low degree of consumption risk sharing between these states and others.

After discussing the magnitude of estimated frictions, we proceed to quantify their impacts by conducting counterfactual analyses where we turn off one friction at a time.

To focus on the impacts of bilateral frictions, we will shut down the frictions between a pair of states denoted as economies 1 and 2 ($x_{12}, x_{21}, x \in \{\tau, d, f\}$) but not the frictions between either state with ROE denoted as economy 3 ($x_{i3}, x_{3i}, i \in \{1, 2\}$). This set of counterfactual analyses consists of three parts. First, we examine the influences of bilateral frictions on bilateral ties in the three channels of risk sharing. Second, we evaluate the effects of frictions on bilateral consumption comovement. Third, we compute the state-level consumption level and volatility averaged across pairs to study the overall effects of these frictions.

Figure 11: Average Friction by State



(a) Trade (b) Migration (c) Finance

This figure plots the estimated frictions averaged across all the state pairs each state forms. A darker color suggests a higher value of frictions. Frictions are calculated as the geometric average of bidirectional frictions in logarithms.

Table 8 reports the key statistics of bilateral trade, migration, and asset shares as

$$\frac{1}{2}(z_{12} + z_{21}) = \frac{1}{2}\left(\frac{Z_{12}}{\sum_{i=1,2,3} Z_{1i}} + \frac{Z_{21}}{\sum_{i=1,2,3} Z_{2i}}\right), \qquad z \in \{\pi, m, \hat{\alpha}\} \qquad Z \in \{X, N, \alpha\}. \quad (45)$$

We calculate these bilateral shares in the original case under the calibrated frictions and in the counterfactual case where corresponding bilateral frictions $x_{12}, x_{21}, x \in \{\tau, d, f\}$ are shut down. Based on the reported results, bilateral economic ties in all the three channels strengthen remarkably under counterfactual scenarios absent bilateral frictions. For example, bilateral trade shares rise from 0.0061 to 0.4411 on average across state pairs when there are no bilateral trade costs. Moreover, the elimination of migration costs raises average bilateral migration shares from 0.0008 to 0.4910, while the elimination of financial frictions raises bilateral asset holdings from 0.1633 to 0.2326. What is common about these counterfactual scenarios is that, these bilateral shares turn out to be close in value to each state's own shares:

$$z_{12} \approx z_{11}, \qquad z \in \{\pi, m, \hat{\alpha}\}. \quad (46)$$

Table 8: Counterfactual Bilateral Ties

	(I). With Friction		(II). Without Friction	
	Mean	Median	Mean	Median
Trade	0.0061	0.0030	0.4411	0.4557
Migration	0.0008	0.0005	0.4910	0.4920
Finance	0.1633	0.1745	0.2326	0.2392

This table reports the counterfactual bilateral shares when corresponding bilateral frictions are turned off. Variables reported include the mean and median values of bilateral trade, migration, and asset shares across all the state pairs in the sample.

The reasoning behind this result is that when a pair of states form an economic zone without barriers, they treat each other like themselves when exchanging goods, labor, and assets. Meanwhile, they drastically cut economic linkages from the rest of the economy with which frictions are considerably higher.

Moreover, we analyze the counterfactual pattern of bilateral consumption comovement. Specifically, table 9 reports the median value of bilateral correlation coefficient of consumption per capita (ρ_c) across the state pairs in the sample. The median correlation of consumption increases from 0.4159 in the benchmark case with calibrated frictions to 0.7354 when there is no trade cost. Meanwhile, the correlation drops to 0.3953 when there is no migration cost and rises to 0.4293 when there is no financial cost. These findings are largely consistent with the two-region analysis from the theory section (see figures 7-9). While the decrease in trade costs inarguably raises consumption correlation, the reduction in migration or financial frictions yields nonmonotonic predictions. In this situation where migration exacerbates cross-region wage inequality following terms-of-trade adjustments, a decline in the migration costs leads to lower consumption correlation. In the financial channel, the influence of financial frictions on consumption correlation depends on whether the magnitude of the frictions is large enough to encourage migration in the direction that worsens wage disparity. The estimated financial frictions are not so high to alter migration in such a way. Hence, eliminating financial frictions facilitates consumption risk sharing, consistent with the classic prediction from Lucas (1982).

Furthermore, we examine the overall impacts of bilateral frictions on state-level consumption. For this purpose, we first compute the steady-state level as well as the volatility of consumption per capita of the two states forming each of the $\frac{\mathcal{I}(\mathcal{I}-1)}{2} = 1225$ state pairs. After that, we take the median value across the $\mathcal{I} - 1 = 49$ state pairs involving a specific state as that state's consumption level and volatility. Table A.5 presents the ratio

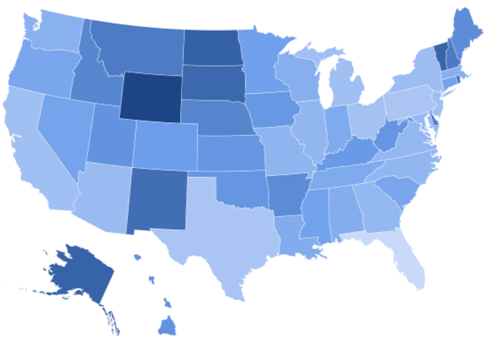
Table 9: Counterfactual Bilateral Consumption Comovement

	Original	No τ	No d	No f
ρ_c	0.4159	0.7354	0.3953	0.4293

This table reports the median bilateral consumption correlation (ρ_c) across state pairs in the original case and when trade costs (τ), migration costs (d), and financial frictions (f) are turned off.

of counterfactual consumption to that in the original case. Based on the reported values, figures 12-13a visualize the counterfactual consumption level compared to that in the original case.[11] Most states witness improvements to their consumption levels when there is no trade cost. In general, states that are subject to the highest trade costs experience the greatest boost in consumption under the counterfactual circumstance. For example, Alaska's consumption rises by 29.8% with the reduction of trade costs. Across the states, the median increase in consumption under the elimination of trade costs is 7.3%.

Figure 12: Counterfactual Consumption without Trade Costs



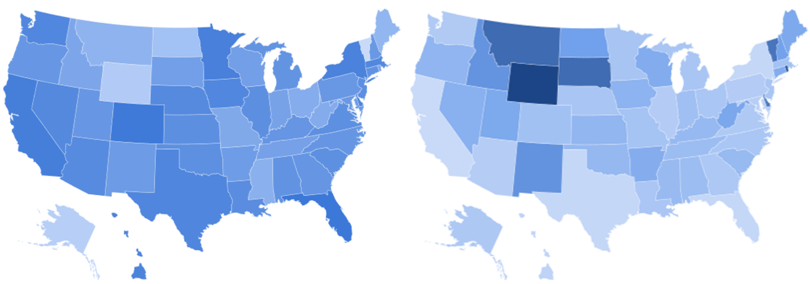
Note: This figure plots the ratio of counterfactual to original level of consumption per capita in the steady state of the economy when bilateral trade cost is shut down. A darker color in the map suggests a higher ratio. Data are reported in table A.5.

The reduction in migration costs, on the other hand, generates a more disparate pattern across states. While the most affluent states including Florida and California benefit from labor mobility, most other states expect lower consumption per capita when the restriction on population is lifted. Across the states, the median change in consumption per capita is -3.2% when migration costs are removed. To explain this pattern, we show the change of each state's population size in figure 13b whose geographic pattern is almost the opposite to figure 13a's: a larger population is associated with a lower consumption per

[11]Since financial frictions are second-order in magnitude and will therefore not affect the consumption level in the steady state of the model, we focus on the situations with no trade and migration costs.

capita. What happens is that the elimination of bilateral migration costs causes drastic population inflows for most states from the rest of the economy (ROE). This is not driven by the change in the migration cost between a state-pair and ROE (i.e. $d_{i3}, d_{3i}, i = 1, 2$) which is assumed to be fixed in this counterfactual analysis, but driven by the change in the migration costs between the pair of states (i.e. d_{12}, d_{21}). Based on the rule of migration decisions (equation 29), households move to a state with a high "option value," which captures the expected future payoff from moving from that state to other states. Therefore, states like Wyoming with the darkest color in figure 13b attract large migration inflows because people find it easier to move from those states to California in the future due to the reduction in their bilateral migration costs with California. Nevertheless, unlike California whose TFP is high enough to benefit from the rise in labor supply, Wyoming does not have enough jobs to meet the needs of the increased population. Consequently, wage declines under the labor market clearing condition, which translates to a lower consumption level. This reasoning explains the disparate pattern of consumption across states generated by the reduction in migration costs in figure 13a. Nevertheless, it is worth noting that our current theoretical framework assumes that consumption is the main driver for migration and therefore neglects other factors including residential land and amenities which also play an important role in migration decisions (see, for example, Saiz (2010), Albouy and Lue (2015), Monte et al. (2018)). Once considered, these factors will dampen households' motives to migrate to states with less elastic housing supply and lower quality of amenities, and reshape the counterfactual population pattern in figure 13b. Despite these considerations, the elimination of migration costs still generates consumption gains for some states and losses for others, largely due to the zero-sum redistribution of population across states caused by it.

Figure 13: Counterfactual Consumption and Population without Migration Costs

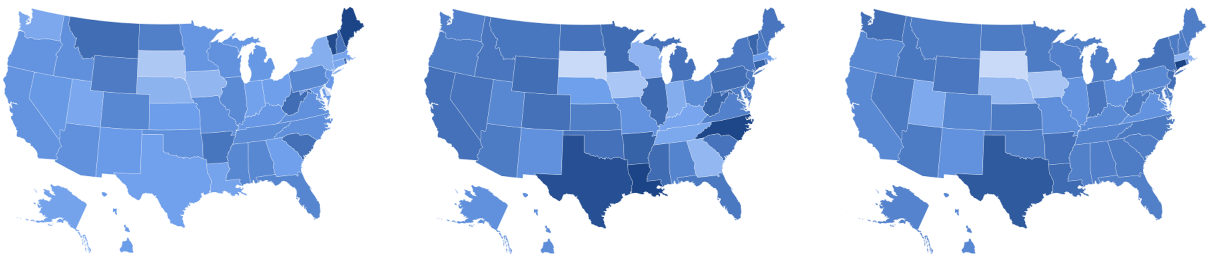


(a) Consumption p.c. (b) Population

Note: This figure plots the ratio of counterfactual to original level of consumption per capita and population in the steady state of the economy when bilateral migration cost is shut down. A darker color in the map suggests a higher ratio. Data are reported in table A.5.

After examining the level of consumption per capita, we continue to investigate its volatility measured as the standard deviation. Figure 14 illustrates the ratio of consumption volatility in the counterfactual case to that in the original case. As is reported in table A.5, the volatility of consumption is lower on average under all the three counterfactual scenarios. The mean reduction in consumption volatility across states is 0.7%, 1.0%, and 0.3% respectively when bilateral trade, migration, and financial frictions are turned off. The magnitude of change is relatively small since it is driven by the elimination of bilateral frictions but not the overall frictions with respect to the rest of the economy. The three plots in figure 14 exhibit geographic resemblance, which implies the substitutability among the channels of risk sharing in lowering consumption volatility of the states most subject to frictions. For a risk-averse agent, lower consumption volatility indicates higher lifetime utility. Therefore, the finding that shutting down the frictions reduces consumption fluctuations reiterates the significance of the three channels of risk sharing for improving welfare.

Figure 14: Counterfactual Volatility of Consumption



(a) No τ (b) No d (c) No f

Note: This figure plots the ratio of counterfactual to original volatility of consumption per capita. A darker color suggests a higher ratio. Data are reported in table A.5.

Last but not least, we use the counterfactual exercises conducted above to deliver policy implications. Our earlier analysis about the spatial characteristics of frictions indicates that eliminating the barriers in the channels of risk sharing can be challenging due to geographic constraints. Nevertheless, macroeconomic policies can be introduced to alleviate the negative impacts of the frictions. In particular, fiscal transfers have been acknowledged as an important channel of risk sharing within a country. Redistribution of wealth from beneficiaries to victims of frictions can potentially undo the influences of frictions on the level and volatility of consumption. On the modeling side, introducing optimal fiscal transfers T_i^* rewrites the wealth constraint of state i

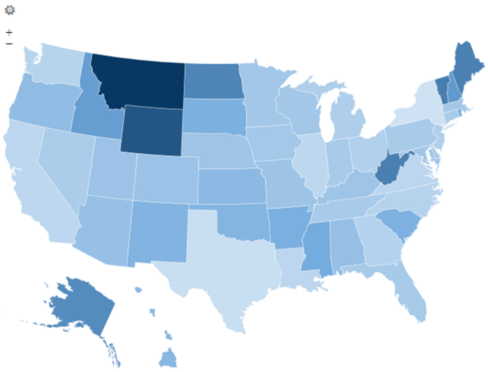
$$\mathcal{W}_{i,t+1} = R_{\mathcal{I},t}\mathcal{W}_{i,t} + \sum_{j}^{\mathcal{I}} \alpha_{j,i,t}(e^{-f_{ji}}R_{j,t} - e^{-f_{\mathcal{I}i}}R_{\mathcal{I},t}) + p_{i,t}\sum_{s} Y_{is,t} + T_{i,t}^* - P_{i,t}C_{i,t} - P_{Ii,t}I_{i,t}, \tag{47}$$

Under the new constraint, households in state i adjust their consumption expenditure and make migration decisions based on the new cross-state consumption differentials. Meanwhile, the portfolio of state i is re-constructed according to the risk-sharing needs under the new wealth constraint. Therefore, the design of fiscal policies requires taking into consideration the endogenous changes of variables in the existing channels of risk sharing and the interplay among these channels.

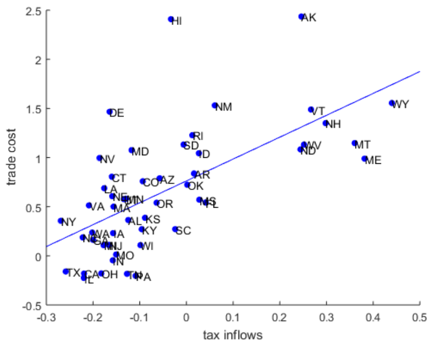
To exemplify such policy analysis, we examine the fiscal transfer that mitigates the reduction of consumption caused by bilateral trade costs. To keep this example simple, we don't impose an aggregate budget constraint for the federal government or restrict the amount of transfers it distributes to any state. The analysis involves the following steps. Step 1, we calculate the policy's targeted moment which is the counterfactual level of consumption when there is no bilateral trade cost. Step 2, we use the state-level wages in the original case as the initial guess, and solve for the supply and demand of labor in each state given the counterfactual trade and migration patterns under no trade cost. After that, we update the value of wages that clear the labor market. We repeat this procedure until the difference between the old and new wages is small enough which pins down the equilibrium wages under the counterfactual scenario. Step 3, we solve for the values of all the endogenous variables in the model based on the wages from step 2 and calculate the corresponding level of consumption. Step 4, we repeat steps 2 and 3 until the model-predicted consumption converges to the targeted moment from step 1. We conduct this analysis for all the state pairs and, for cross-state comparison plot the median tax transfers across the state pairs formed by each state in figure 15. It illustrates that regions that are estimated to be confronted with higher trade costs, such as Wyoming, Montana, and Alaska, should receive more tax transfers to mitigate the impacts of trade frictions on their consumption. In contrast, states that face lower trade costs, including New York, Texas, and California, should be net tax payers to achieve the counterfactual outcome. The general relationship between the predicted transfers and the estimated trade costs is positive.

This example shows that the quantitative model we propose in this paper provides a useful framework for policy analyses. The framework is general enough to accommodate

Figure 15: Tax Transfers under Trade Costs



(a) Predicted Transfer Inflows



(b) Estimated τ and Transfers

Note: Figure (a) plots the tax transfers as shares of a state's GSP to achieve its level of consumption in the counterfactual situation absent trade costs. A darker color in the heatmap suggests more tax inflows. Figure (b) shows the positive relationship between the transfer and estimated trade costs, calculated as the geometric mean of inbound and outbound trade costs reported in table A.4. These costs are positively correlated with the predicted tax transfers at the 1% level of significance.

a rich set of targeted moments including the level, volatility, and covariance of macroeconomic variables. Meanwhile, the framework is flexible enough to be adapted to any other country under the specific budget constraint its government is subject to. These policies, if well designed and implemented, facilitate risk sharing and hence reduce both consumption volatility over time and consumption disparity across regions.

5 Conclusion

This paper explores the role of bilateral economic exchanges influenced by geography in shaping the pattern of consumption across fifty states in the US. Failure of consumption risk sharing has been recognized as a major puzzle in the macroeconomic literature. To explain this puzzle, our research exploits variations among state pairs and analyzes frictions that dampen bilateral consumption comovement. Specifically, we propose a comprehensive and unified approach that encompasses trade, migration, and finance as channels of consumption risk sharing.

In the paper we first empirically establish a gravity model by documenting that bilateral risk sharing decreases in geographic distance among the US states. To explain this fact, we develop a theoretical model to explore the impacts of frictions in the channels of risk sharing that potentially covary with distance. We start with a two-economy framework following Backus et al. (1992) to examine the mechanism of different channels affect consumption as well as how they interact with each other. After that, we extend the model to a multi-region framework calibrated to the US data for a quantitative assessment. The framework enables us to quantify not only the magnitude but also the influence of each friction through counterfactual analysis. The quantitative framework also serves as a useful tool for the design of macroeconomic policies.

One important extension of our real business cycle (RBC) framework is to introduce the New Keynesian ingredients including nominal rigidity. As is pointed out by Crucini et al. (2010), both nominal rigidities and trade costs (proxied by distance) generate price disparities within a country. Besides, Hazell et al. (2022) reason that even within a monetary union, cross-region heterogeneity generates different slopes of the Phillips Curve under a uniform national monetary policy, which consequently creates welfare disparity across economies. Therefore, extensions of our model can incorporate monetary factors into the analysis when explaining cross-state consumption comovement. On the other hand, the frictional economic linkages examined by our model, in particular through the channels of finance and migration, are absent in their analysis. Hence, our paper complements that literature by accounting for the transmission and propagation of economic shocks through disaggregate cross-region economic linkages. Other papers pursuing this research direction include House et al. (2018) and House et al. (2020), who quantify the welfare outcome of micro-founded economic exchanges under monetary policies.

Our paper focuses on cross-state risk sharing within the US as an example, but our framework is general and flexible enough to be tailored to another context of interest. Therefore, we can apply the framework to explain consumption synchronization not only within but also across countries,[12] or even both simultaneously. For example, one application of the model is to compare intra- and inter-national linkages to diagnose the border effects of risk sharing proposed by Devereux and Hnatkovska (2020), who document a sharp decrease of consumption comovement at the US-Canada border. By quantifying the level and influence of frictions in this setting, our framework can provide guidance for trade and exchange rate policies with a target to reduce consumption disparity and raise social welfare both within and across country borders.

[12] In particular, our framework is useful to be employed in such a context as the European Union and NAFTA where bilateral exchanges in different channels are commonplace.

References

Albouy, D. and Lue, B. Driving to opportunity: Local rents, wages, commuting, and sub-metropolitan quality of life. *Journal of Urban Economics*, 89:74–92, 2015.

Anderson, J. E. and Van Wincoop, E. Gravity with gravitas: A solution to the border puzzle. *American Economic Review*, 93(1):170–192, 2003.

Armington, P. S. A theory of demand for products distinguished by place of production. *Staff Papers*, 16(1):159–178, 1969.

Artuc, E., Chaudhuri, S., and McLaren, J. Trade shocks and labor adjustment: A structural empirical approach. *American Economic Review*, 100(3):1008–45, 2010.

Asdrubali, P., Sorensen, B., and Yosha, O. Channels of interstate risk sharing: United states 19631990. *The Quarterly Journal of Economics*, 111(4):1081–1110, 1996.

Athanasoulis, S. G. and Van Wincoop, E. v. Risk sharing within the united states: What do financial markets and fiscal federalism accomplish? *Review of Economics and Statistics*, 83(4):688–698, 2001.

Aviat, A. and Coeurdacier, N. The geography of trade in goods and asset holdings. *Journal of International Economics*, 71(1):22–51, 2007.

Backus, D. K., Kehoe, P. J., and Kydland, F. E. International real business cycles. *Journal of Political Economy*, 100(4):745–775, 1992.

Baxter, M. and Crucini, M. J. Business cycles and the asset structure of foreign trade. *International Economic Review*, 36(4):821–854, 1995.

Bems, R. Aggregate investment expenditures on tradable and nontradable goods. *Review of Economic Dynamics*, 11(4):852–883, 2008.

Caliendo, L., Parro, F., Rossi-Hansberg, E., and Sarte, P.-D. The impact of regional and sectoral productivity changes on the us economy. *The Review of Economic Studies*, 85(4):2042–2096, 2018.

Chertman, F., Choo, D., and Hu, C. Trade costs and a gravity model of risk sharing. 2020.

Cole, H. L. and Obstfeld, M. Commodity trade and international risk sharing: How much do financial markets matter? *Journal of Monetary Economics*, 28(1):3–24, 1991.

Corsetti, G., Dedola, L., and Leduc, S. International risk sharing and the transmission of productivity shocks. *The Review of Economic Studies*, 75(2):443–473, 2008.

Coval, J. D. and Moskowitz, T. J. Home bias at home: Local equity preference in domestic portfolios. *The Journal of Finance*, 54(6):2045–2073, 1999.

Crucini, M. J. On international and national dimensions of risk sharing. *Review of Economics and Statistics*, 81(1):73–84, 1999.

Crucini, M. J., Shintani, M., and Tsuruga, T. The law of one price without the border: the role of distance versus sticky prices. *The Economic Journal*, 120(544):462–480, 2010.

Del Negro, M. Aggregate risk sharing across us states and across european countries. *Yale University*, 1998.

Del Negro, M. Asymmetric shocks among us states. *Journal of International Economics*, 56(2):273–297, 2002.

Devereux, M. B. and Hnatkovska, V. V. Borders and nominal exchange rates in risk-sharing. *Journal of the European Economic Association*, 18(3):1238–1283, 2020.

Devereux, M. B. and Sutherland, A. Country portfolios in open economy macro-models. *Journal of the European Economic Association*, 9(2):337–369, 2011.

Dumas, B. and Uppal, R. Global diversification, growth, and welfare with imperfectly integrated markets for goods. *The Review of Financial Studies*, 14(1):277–305, 2001.

Eaton, J. and Kortum, S. Technology, geography, and trade. *Econometrica*, 70(5):1741–1779, 2002.

Eaton, J., Kortum, S., and Neiman, B. Obstfeld and rogoff? s international macro puzzles: a quantitative assessment. *Journal of Economic Dynamics and Control*, 72:5–23, 2016.

Fitzgerald, D. Trade costs, asset market frictions, and risk sharing. *American Economic Review*, 102(6): 2700–2733, 2012.

Garofalo, G. A. and Yamarik, S. Regional convergence: Evidence from a new state-by-state capital stock series. *Review of Economics and Statistics*, 84(2):316–323, 2002.

Hazell, J., Herreño, J., Nakamura, E., and Steinsson, J. The slope of the phillips curve: evidence from us states. *Quarterly Journal of Economics*, 2022.

Heathcote, J. and Perri, F. Financial globalization and real regionalization. *Journal of Economic Theory*, 119(1):207–243, 2004.

Heathcote, J. and Perri, F. The international diversification puzzle is not as bad as you think. *Journal of Political Economy*, 121(6):1108–1159, 2013.

Heathcote, J., Storesletten, K., and Violante, G. L. Consumption and labor supply with partial insurance: An analytical framework. *American Economic Review*, 104(7):2075–2126, 2014.

Hess, G. D. and Shin, K. Intranational business cycles in the united states. *Journal of International Economics*, 44(2):289–313, 1998.

House, C. L., Proebsting, C., and Tesar, L. L. Quantifying the benefits of labor mobility in a currency union. Technical report, National Bureau of Economic Research, 2018.

House, C. L., Proebsting, C., and Tesar, L. L. Regional effects of exchange rate fluctuations. *Journal of Money, Credit and Banking*, 52(S2):429–463, 2020.

Isard, W. Location theory and trade theory: short-run analysis. *The Quarterly Journal of Economics*, pages 305–320, 1954.

Johnson, N. N. Tradable and nontradable inflation indexes: replicating new zealands tradable indexes with bls cpi data. *Monthly Labor Review*, pages 1–25, 2017.

Kalemli-Ozcan, S., Sorensen, B. E., and Yosha, O. Risk sharing and industrial specialization: Regional and international evidence. *American Economic Review*, 93(3):903–918, 2003.

Kalemli-Ozcan, S., Reshef, A., Sørensen, B. E., and Yosha, O. Why does capital flow to rich states? *The Review of Economics and Statistics*, 92(4):769–783, 2010.

Kollmann, R. Consumption, real exchange rates and the structure of international asset markets. *Journal of International Money and Finance*, 14(2):191–211, 1995.

Kose, M. A., Prasad, E. S., and Terrones, M. E. Does financial globalization promote risk sharing? *Journal of Development Economics*, 89(2):258–270, 2009.

Lewer, J. J. and Van den Berg, H. A gravity model of immigration. *Economics letters*, 99(1):164–167, 2008.

Lewis, K. K. What can explain the apparent lack of international consumption risk sharing? *Journal of Political Economy*, 104(2):267–297, 1996.

Lucas, R. E. Interest rates and currency prices in a two-country world. *Journal of Monetary Economics*, 10(3):335–359, 1982.

Martin, P. and Rey, H. Financial super-markets: size matters for asset trade. *Journal of International Economics*, 64(2):335–361, 2004.

Mendoza, E. G. Real business cycles in a small open economy. *American Economic Review*, pages 797–818, 1991.

Monte, F., Redding, S. J., and Rossi-Hansberg, E. Commuting, migration, and local employment elasticities. *American Economic Review*, 108(12):3855–90, 2018.

Nakamura, E. and Steinsson, J. Fiscal stimulus in a monetary union: Evidence from us regions. *American Economic Review*, 104(3):753–92, 2014.

Obstfeld, M. and Rogoff, K. The six major puzzles in international macroeconomics: is there a common cause? *NBER macroeconomics annual*, 15:339–390, 2000.

Okawa, Y. and Van Wincoop, E. Gravity in international finance. *Journal of International Economics*, 87(2):205–215, 2012.

Parsley, D. and Popper, H. Risk sharing in a politically divided monetary union. *Open Economies Review*, 32(4):649–669, 2021.

Portes, R. and Rey, H. The determinants of cross-border equity flows. *Journal of International Economics*, 65(2):269–296, 2005.

Ramos, R. and Suriñach, J. A gravity model of migration between the enc and the eu. *Tijdschrift voor economische en sociale geografie*, 108(1):21–35, 2017.

Redding, S. J. and Rossi-Hansberg, E. Quantitative spatial economics. *Annual Review of Economics*, 9: 21–58, 2017.

Saiz, A. The geographic determinants of housing supply. *The Quarterly Journal of Economics*, 125(3): 1253–1296, 2010.

Simonovska, I. and Waugh, M. E. The elasticity of trade: Estimates and evidence. *Journal of International Economics*, 92(1):34–50, 2014.

Storesletten, K., Telmer, C. I., and Yaron, A. Consumption and risk sharing over the life cycle. *Journal of Monetary Economics*, 51(3):609–633, 2004.

Tille, C. and Van Wincoop, E. International capital flows. *Journal of International Economics*, 80(2): 157–175, 2010.

Tinbergen, J. Shaping the world economy; suggestions for an international economic policy. 1962.

Figure A.1: U.S. Map

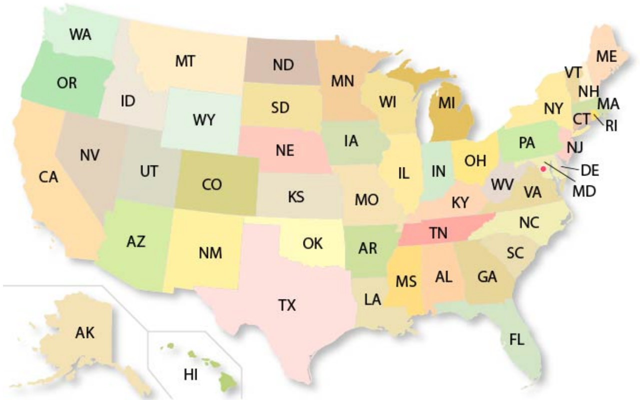


Table A.1: List of US States with Abbreviations

Name	Abbreviation	Name	Abbreviation	Name	Abbreviation	Name	Abbreviation	Name	Abbreviation
Alabama	AL	Hawaii	HI	Massachusetts	MA	New Mexico	NM	South Dakota	SD
Alaska	AK	Idaho	ID	Michigan	MI	New York	NY	Tennessee	TN
Arizona	AZ	Illinois	IL	Minnesota	MN	North Carolina	NC	Texas	TX
Arkansas	AR	Indiana	IN	Mississippi	MS	North Dakota	ND	Utah	UT
California	CA	Iowa	IA	Missouri	MO	Ohio	OH	Vermont	VT
Colorado	CO	Kansas	KS	Montana	MT	Oklahoma	OK	Virginia	VA
Connecticut	CT	Kentucky	KY	Nebraska	NE	Oregon	OR	Washington	WA
Delaware	DE	Louisiana	LA	Nevada	NV	Pennsylvania	PA	West Virginia	WV
Florida	FL	Maine	ME	New Hampshire	NH	Rhode Island	RI	Wisconsin	WI
Georgia	GA	Maryland	MD	New Jersey	NJ	South Carolina	SC	Wyoming	WY

Appendices

A Figures and Tables

Tables A.2 and A.3 report the results of two sets of robustness checks for the gravity model of risk sharing. First, we consider alternative data sources for state-level consumption and inflation, and for bilateral geographic distance. Second, we reconstruct measures of bilateral risk sharing after adjusting for additional time-series and cross-section variations (see the detailed description in the next paragraph). The results reported in the tables suggest that our finding about the comovement between geographic distance and consumption risk sharing remains robust.

When constructing alternative measures of bilateral risk sharing, first we consider demographic variables whose dynamics potentially shift consumption demand over time. These state-level variables (denoted as $X_{i,t}$) include average age, gender ratio, and education levels, whose data are obtained from the American Community Survey conducted by the Census Bureau. The estimation of risk sharing coefficients becomes

$$\Delta \log\ c_{it} - \Delta \log\ c_{jt} = \alpha_{ij} + \beta_{ij}(\Delta \log\ y_{it} - \Delta \log\ y_{jt}) + \mu_i X_{i,t} + \mu_j X_{j,t} + \epsilon_{ijt}. \quad \text{(A1)}$$

Second, we adjust for states' distinct exposure to aggregate risks when measuring bilateral

Table A.2: Spatial Pattern of Risk Sharing – Alternative Data Sources

Dep. Var.: $\hat{\beta}_{ij}$	A. Alternative Price		B. Alternative Consumption		C. Alternative Distance	
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(d_{ij})$	0.119 ***	0.176 ***	0.041 ***	0.050 ***	0.151 ***	0.211 ***
	(0.017)	(0.024)	(0.005)	(0.007)	(0.010)	(0.012)
Geographic Variables	N	Y	N	Y	N	Y
Political Dissimilarity	N	Y	N	Y	N	Y
Industrial Dissimilarity	N	Y	N	Y	N	Y
Observations	528	528	1225	1225	1225	1225
R^2	0.077	0.183	0.056	0.148	0.161	0.288

Robust standard errors in parentheses. *** significant at 1%. The dependent variable is the estimated risk sharing coefficient $\hat{\beta}_{ij}$. d_{ij} denotes the geographic distance between state i and j. Panel A uses the state-level CPI data by Hazell et. al. (2020), Panel B uses the BEA consumption data, and Panel C uses the shipment distance from the CFS. Geographic variables and political/industrial dissimilarity measures remain the same as in the baseline estimation (table 2).

risk sharing, as the difference in output growth between a pair of states in equation 1 may reflect the two states' heterogeneous exposure to national shocks. To address this potential mismeasurement of local output shocks, we first estimate β_i and β_j from

$$\Delta \log\ y_{it} = \alpha_i + \beta_i \Delta \log\ y_{USt} + \epsilon_{it}, \qquad \Delta \log\ y_{jt} = \alpha_j + \beta_j \Delta \log\ y_{USt} + \epsilon_{jt}, \tag{A2}$$

where $\Delta \log\ y_{USt}$ denotes the growth of log real per-capita output of the US, and hence β_i captures the impact of aggregate shocks on state i's output. We then estimate β_{ij} from

$$\Delta \log\ c_{it} - \Delta \log\ c_{jt} = \alpha_{ij} + \beta_{ij}[(\Delta \log\ y_{it} - \beta_i \Delta \log\ y_{USt}) - (\Delta \log\ y_{jt} - \beta_j \Delta \log\ y_{USt})] + \epsilon_{ijt}. \tag{A3}$$

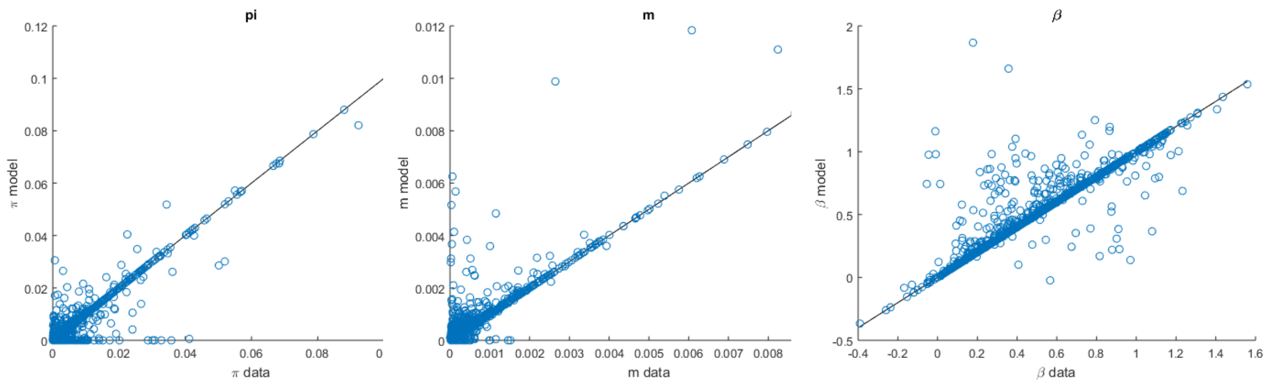
We also consider the bootstrap method for the potential finite sample bias from equation A2. Specifically, we draw a random sample with replacement (30 out of 43 years of sample) when running regression A3 to generate β_{ij}. When we regress the obtained β_{ij} on distance 1000 times for its estimate γ, we find the result to remain significant at 1% given the confidence interval as the 2.5% and 97.5% quantiles of the $\hat{\gamma}$ distribution.

Table A.3: Spatial Pattern of Risk Sharing – Alternative β

Dep. Var.: $\hat{\beta}_{ij}$	A. β_{ij} adjusted for demand shifters		B. β_{ij} adjusted for aggregate shocks	
	(1)	(2)	(3)	(4)
$\log(d_{ij})$	0.128 ***	0.143 ***	0.147 ***	0.214 ***
	(0.013)	(0.017)	(0.010)	(0.012)
Geographic Variables	N	Y	N	Y
Political Dissimilarity	N	Y	N	Y
Industrial Dissimilarity	N	Y	N	Y
Observations	1225	1225	1225	1225
R^2	0.067	0.205	0.148	0.315

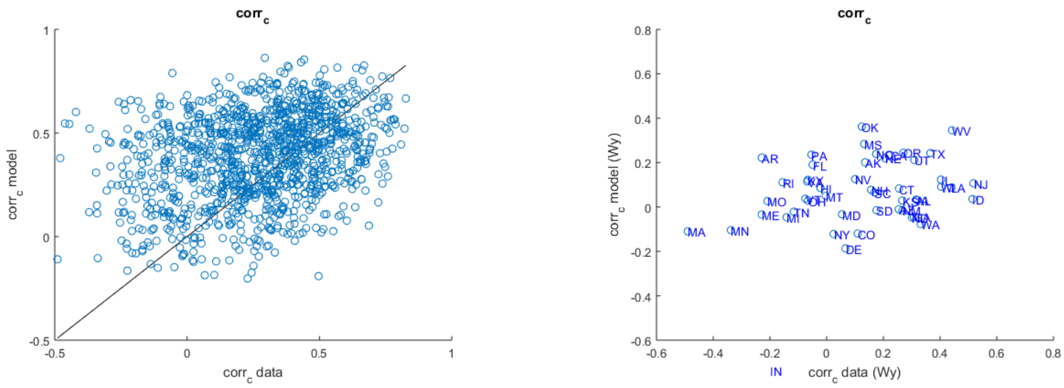
Robust standard errors in parentheses. *** significant at 1%. The dependent variable in panel A (B) is the estimated β_{ij} based on equation A1 (A3). d_{ij} denotes distance between i and j. Geographic variables and political/industrial dissimilarity measures remain the same as in the baseline estimation (table 2).

Figure A.2: Model Fit (I)



Note: This figure plots the relationship between model-implied and actual bilateral ties including bilateral trade shares, bilateral migration shares, and coefficients of consumption risk sharing. Empirical moments are on the horizontal axis, and theoretical moments are on the vertical axis. The plots show the model performs well in matching these empirical moments.

Figure A.3: Model Fit (II)



Note: This figure plots the relationship between model-implied and actual bilateral consumption correlations. Empirical moments are on the horizontal axis, and theoretical moments are on the vertical axis. The left diagram covers all the state pairs, the right covers the pairs formed by Wyoming as an example.

Table A.4: Estimated Frictions by State

	Trade Cost		Migration Cost		Financial Cost	
State	Outbound	Inbound	Outbound	Inbound	Outbound	Inbound
AL	0.975	1.476	1.035	1.117	0.493	0.592
AK	3.136	3.643	0.888	1.146	30.850	54.888
AZ	1.561	1.410	0.996	0.974	1.403	1.281
AR	1.007	2.296	1.002	1.115	1.562	0.754
CA	1.845	0.452	1.018	0.858	0.930	0.568
CO	1.406	1.520	0.934	0.966	1.379	1.864
CT	1.478	1.513	1.033	1.165	5.474	3.356
DE	1.536	2.822	1.069	1.175	80.416	72.842

FL	1.731	0.994	1.007	0.821	1.277	7.177
GA	1.057	1.113	0.970	0.973	1.292	1.393
HI	2.710	4.099	0.980	1.086	6.792	9.723
ID	1.045	2.719	1.019	1.159	3.249	5.006
IL	1.111	0.719	0.988	0.983	0.750	0.672
IN	0.917	1.042	0.999	1.044	3.381	2.784
IA	0.646	1.952	1.005	1.080	7.757	4.730
KS	0.702	2.099	0.978	1.060	3.390	2.600
KY	0.884	1.483	1.000	1.074	7.201	6.939
LA	1.151	1.729	1.030	1.105	2.384	3.223
ME	1.128	2.384	1.019	1.181	0.002	2.119
MD	1.766	1.660	1.029	1.058	9.218	3.651
MA	1.374	1.200	1.005	1.064	3.732	3.272
MI	0.938	1.189	1.030	1.038	2.645	4.517
MN	1.150	1.555	1.025	1.076	1.414	0.780
MS	0.865	2.047	1.014	1.153	2.014	6.122
MO	0.921	1.101	1.008	1.032	1.119	0.827
MT	1.291	2.440	0.975	1.152	0.022	0.201
NE	1.082	1.695	1.025	1.167	14.183	14.576
NV	1.319	2.052	0.980	1.086	1.060	1.493
NH	1.522	2.535	1.013	1.193	1.580	3.732
NJ	1.012	1.104	1.018	1.068	0.899	0.883
NM	2.197	2.103	0.998	1.128	8.109	14.685
NY	2.122	0.673	1.074	0.977	8.658	7.305
NC	0.901	1.339	1.018	0.957	0.646	0.924
ND	0.910	3.245	0.984	1.177	0.735	5.364
OH	0.943	0.887	1.030	1.027	0.708	0.607
OK	1.077	1.913	1.036	1.113	1.754	0.808
OR	1.083	1.585	1.027	1.128	3.052	3.060
PA	1.070	0.762	1.021	1.032	0.216	0.308
RI	1.081	3.156	1.068	1.213	0.690	1.087
SC	0.983	1.334	1.003	1.034	0.283	0.633
SD	0.909	3.413	0.997	1.162	11.196	11.012
TN	0.884	0.942	0.978	0.995	1.836	2.071
TX	1.236	0.690	0.999	0.849	1.249	1.208
UT	0.951	1.873	1.013	1.125	2.752	3.114
VT	1.082	4.098	1.035	1.214	0.023	0.374
VA	1.252	1.335	0.997	0.976	2.416	2.006
WA	0.954	1.330	1.018	1.006	1.188	1.222
WV	1.070	2.900	1.084	1.201	0.308	14.961
WI	1.166	0.957	1.037	1.082	0.926	0.692
WY	1.490	3.177	0.932	1.157	0.018	0.566

This table presents the normalized trade, migration, and financial costs averaged across state pairs for each state. Step 1, we calculate both inbound and outbound frictions averaged across $\mathcal{I}-1$ pairs a state i forms: ($x_i^{ex} = \text{mean}(x_{ij}),\ x_i^{in} = \text{mean}(x_{ji}), j \neq i \in [1, \mathcal{I}], x \in \{\tau, d, f\}$). Step 2, we normalize the average friction of Georgia and Ohio, the median states in terms of output per capita, to 1 in each channel: $x^{ex}_{\text{GA,OH}} = x^{in}_{\text{GA,OH}} = 1$. We report the ratio of state-level frictions from step 2 to the median states' in the table for cross-state comparison.

Table A.5: Counterfactual Consumption Relative to Benchmark

	Equilibrium Level $\bar{c}$		Volatility σ_c		
State	No τ	No d	No τ	No d	No f
AL	1.058	0.958	1.015	0.974	1.007
AK	1.298	0.955	0.908	0.969	0.981
AZ	1.072	0.985	0.999	0.993	1.000
AR	1.161	0.981	1.068	1.015	1.000
CA	1.033	1.044	0.987	1.047	1.018
CO	1.067	0.978	1.036	1.009	1.049
CT	1.092	0.998	0.939	1.006	0.993
DE	1.202	0.967	0.816	0.970	0.998
FL	0.979	1.032	0.998	0.995	1.003
GA	1.026	0.983	0.966	0.971	1.003
HI	1.094	0.977	0.953	0.979	1.000
ID	1.200	0.931	1.036	0.987	1.002
IL	1.009	0.978	0.972	0.994	1.002
IN	1.050	0.943	0.970	0.982	0.958
IA	1.064	0.947	0.879	0.967	1.010
KS	1.059	0.962	0.959	0.963	0.986
KY	1.051	0.948	0.966	0.983	0.998
LA	1.075	0.968	0.897	1.002	0.991
ME	1.165	0.939	1.156	0.971	1.000
MD	1.070	0.974	1.003	0.990	1.001
MA	1.036	0.980	0.958	0.988	1.004
MI	1.021	0.993	0.958	0.999	1.005
MN	1.082	0.972	0.997	0.966	1.006
MS	1.127	0.954	1.033	0.990	0.991
MO	1.071	0.974	1.022	0.992	0.990
MT	1.213	0.906	1.112	0.985	1.000
NE	1.136	0.957	0.910	1.017	0.965
NV	1.097	0.968	0.979	0.985	1.000
NH	1.250	0.983	1.106	0.992	1.000
NJ	1.002	0.976	0.946	0.990	1.001
NM	1.221	0.988	0.969	1.018	0.996
NY	1.027	1.018	0.956	1.038	1.000
NC	1.024	0.989	0.975	1.004	0.969
ND	1.263	0.919	1.041	1.032	1.000
OH	1.014	0.965	0.957	1.010	1.010
OK	1.080	0.964	0.997	0.984	0.981
OR	1.070	0.952	0.982	0.977	0.959
PA	1.001	0.974	1.012	0.987	1.000
RI	1.197	0.946	1.117	0.984	1.007
SC	1.091	0.959	1.08	0.965	1.003
SD	1.245	0.903	0.901	0.928	0.951
TN	1.075	0.955	0.999	0.981	1.000
TX	0.964	0.993	0.932	1.031	1.032
UT	1.135	0.962	0.971	0.979	0.995
VT	1.329	0.909	1.193	0.985	1.000
VA	1.001	0.979	0.999	0.994	1.000
WA	1.033	0.989	0.923	1.005	1.001
WV	1.093	0.941	1.070	1.001	1.004

WI	1.072	0.959	1.030	0.983	0.998
WY	1.356	0.927	1.018	0.962	1.000
Mean	1.103	0.966	0.993	0.990	0.997
Median	1.073	0.968	0.984	0.988	0.999

This table presents each state's median counterfactual steady-state level and volatility of consumption across its state pairs, as a ratio to the values in original case with frictions calibrated to the data. Counterfactual scenarios include the cases absent bilateral trade costs (τ), migration costs (d), and financial frictions (f).

B Data

B.1 State-level output, consumption, and price

The US Bureau of Economic Analysis (BEA) reports the real GDP by state (GSP) since 1977, with data from 1977-1997 reported in the Standard Industrial Classification (SIC) and those from 1997-2019 in the North American Industry Classification (NAICS). To address this discontinuity, we first calculate the annual growth rate based on the SIC-based real GSP, and then reconstruct the time series of real GSP from 1977 to 1997 using this annual growth rate and the NAICS-based real GSP in 1997.

The nominal consumption data from the BEA are only available after 1997, which is not ideal for our risk-sharing analysis over a long horizon. Therefore, we follow Asdrubali et al. (1996)'s method of constructing state-level private consumption by rescaling state-level retail sales by the country-level ratio of private consumption to retail sales, both obtained from the BEA. To convert nominal to real consumption, we use the state-level inflation series constructed by Nakamura and Steinsson (2014) over the period from 1966 to 2008. They obtain the inflation series from 1966 to 1995 from Del Negro (1998), who combines the BLS regional inflation data and cost-of-living estimates from the American Chamber of Commerce Realtors Association (ACCRA). For the estimates between 1995 and 2008, they multiply a population-weighted average of cost-of-living indices from the ACCRA across states with the US aggregate CPI. After 2008, we use the Regional Price Parities (RPP) from the BEA that measure price differences within the United States. RPP is a weighted average of the price level of goods and services for the average consumer in one geographic region compared to all other regions in the US. We merge these data to construct a state-level CPI index for 1966-2019, using which we deflate the nominal consumption data to calculate real consumption at the state level.

We also use alternative data sources to verify the robustness of the gravity model. Table A.2 Panel A uses state-level inflation from Hazell et al. (2022) who construct CPI with micro data gathered by the BLS from 1978 to 2017. Panel B uses only the recent BEA data of consumption expenditure and real GSP between 1997 and 2018.

B.2 Bilateral trade and migration flows

The Commodity Flow Survey (CFS) is conducted every five years by the Census Bureau in partnership with the Department of Transportation. The survey provides detailed information on commodity flows within the US, including the type of commodities shipped, origin and destination, value and weight, and mode of transport. There are six waves of surveys so far (1993, 1997, 2002, 2007, 2012, 2017).

State-to-state migration data are based on year-to-year address changes reported on individual income tax returns filed with the Internal Revenue Service (IRS). Specifically, we use the reported number of returns filed every year to track migration across states. The data are available for filing years 1991 through 2019.

B.3 State-level productivity

We estimate the state-level total factor productivity (TFP) as the Solow residual from

$$\log(A_{i,t}) = \log(Y_{i,t}) - \alpha \log(K_{i,t}) - (1-\alpha)\log(L_{i,t}), \tag{A4}$$

where $Y_{i,t}$, $K_{i,t}$, and $L_{i,t}$ are output, capital, and labor in state i at time t respectively, while α denotes capital share in production. We estimate $1-\alpha$ to be 0.59 by dividing the labor earnings by the economic output based on the BEA data.[13] Moreover, we use the GSP and employment data reported by the BEA for $Y_{i,t}$ and $K_{i,t}$ over the period 1977-2019 for the estimation.

We construct the estimates for state-level capital stock following Garofalo and Yamarik (2002). Namely, we apportion the national private capital stock, to states using sectoral income data from the BEA: For each two-digit NAICS industry

$$K_{i,t}^s = \left(\frac{Y_{i,t}^s}{Y_{US,t}^s}\right) K_{US,t}^s, \tag{A5}$$

where $K_{i,t}^s$ ($Y_{i,t}^s$) refers to capital (output) of industry s in state i at time t, while $K_{US,t}^s$ ($Y_{US,t}^s$) represents country-level capital (output). Each state's capital stock estimate, $K_{i,t}$, is then the sum of sectoral-level capital stock:

$$K_{i,t} = \sum_{s=1}^{S} K_{i,t}^s. \tag{A6}$$

After obtaining the values of all the elements that appear in equation A4, we calculate the state-level TFP with which we subsequently estimate the joint productivity process across states.

[13]The BEA reports the data of labor earning(SAINC5), which consists of compensation of employees and proprietors' income with inventory valuation adjustment and capital consumption adjustment.

C Portfolio Choice in Trilateral Framework

In this section I describe and solve the portfolio choice problem introduced in the theory section within a framework with three economies numbered $i = 1, 2, 3$. Each economy's financial asset, which is its claims to capital income net of investment expenditure, can be traded in an integrated financial market. Nevertheless, there are bilateral financial frictions modeled as transaction costs f_{ij} on returns R_i when j holds assets from i. These second-order frictions appear in the Euler equations of the three economies

$$\begin{aligned}
E_t[\tfrac{U'(c_{1,t+1})}{P_{1,t+1}}R_{1,t+1}] &= E_t[\tfrac{U'(c_{1,t+1})}{P_{1,t+1}}e^{-f_{21}}R_{2,t+1}] &= E_t[\tfrac{U'(c_{1,t+1})}{P_{1,t+1}}e^{-f_{31}}R_{3,t+1}],\\
E_t[\tfrac{U'(c_{2,t+1})}{P_{2,t+1}}R_{2,t+1}] &= E_t[\tfrac{U'(c_{2,t+1})}{P_{2,t+1}}e^{-f_{12}}R_{1,t+1}] &= E_t[\tfrac{U'(c_{2,t+1})}{P_{2,t+1}}e^{-f_{32}}R_{3,t+1}],\\
E_t[\tfrac{U'(c_{3,t+1})}{P_{3,t+1}}R_{3,t+1}] &= E_t[\tfrac{U'(c_{3,t+1})}{P_{3,t+1}}e^{-f_{13}}R_{1,t+1}] &= E_t[\tfrac{U'(c_{3,t+1})}{P_{3,t+1}}e^{-f_{23}}R_{2,t+1}].
\end{aligned} \tag{A7}$$

We derive portfolios with Devereux and Sutherland (2011)'s method by evaluating these Euler equations. First we assume assets from economy 3 to be a numeraire asset and denote the vector of excess returns to the other assets as R_x:

$$\hat{R}'_{x,t} = [\hat{R}_{1,t} - \hat{R}_{3,t}, \hat{R}_{2,t} - \hat{R}_{3,t}], \tag{A8}$$

where $\hat{y}_t$ represents the log-deviation of any variable y from its steady state at t. Next we evaluate the second-order Taylor expansion of the Euler equations as

$$\begin{aligned}
E_t[\hat{R}_{x,t+1} + \tfrac{1}{2}\hat{R}^2_{x,t+1} - (\sigma\hat{c}_{1,t+1} + \hat{P}_{1,t+1})\hat{R}_{x,t+1}] &= -\tfrac{1}{2}\begin{bmatrix} f_{31} \\ f_{31} - f_{21} \end{bmatrix} + \mathcal{O}(\epsilon^3),\\
E_t[\hat{R}_{x,t+1} + \tfrac{1}{2}\hat{R}^2_{x,t+1} - (\sigma\hat{c}_{2,t+1} + \hat{P}_{2,t+1})\hat{R}_{x,t+1}] &= -\tfrac{1}{2}\begin{bmatrix} f_{32} - f_{12} \\ f_{32} \end{bmatrix} + \mathcal{O}(\epsilon^3),\\
E_t[\hat{R}_{x,t+1} + \tfrac{1}{2}\hat{R}^2_{x,t+1} - (\sigma\hat{c}_{3,t+1} + \hat{P}_{3,t+1})\hat{R}_{x,t+1}] &= -\tfrac{1}{2}\begin{bmatrix} -f_{13} \\ -f_{23} \end{bmatrix} + \mathcal{O}(\epsilon^3).
\end{aligned} \tag{A9}$$

where $\hat{R}^{2'}_{x,t+1}$ denotes differences in squared changes of returns

$$\hat{R}^{2'}_{x,t+1} = [\hat{R}^2_{1,t+1} - \hat{R}^2_{3,t+1}, \hat{R}^2_{2,t+1} - \hat{R}^2_{3,t+1}]. \tag{A10}$$

On the right-hand side of equations A9 are vectors of financial frictions each country incurs when holding assets from economies 1 and 2 relative to the frictions associated with its holding assets from economy 3. Plus, the last term $\mathcal{O}(\epsilon^3)$ captures all terms of order higher than two. Taking the difference among equations A9 yields

$$\begin{aligned}
E_t[(\hat{c}_{12,t+1} + \tfrac{\hat{P}_{12,t+1}}{\sigma})\hat{R}_{x,t+1}] &= \tfrac{1}{2\sigma}\begin{bmatrix} f_{31} - f_{32} + f_{12} \\ f_{31} - f_{21} - f_{32} \end{bmatrix} + \mathcal{O}(\epsilon^3),\\
E_t[(\hat{c}_{13,t+1} + \tfrac{\hat{P}_{13,t+1}}{\sigma})\hat{R}_{x,t+1}] &= \tfrac{1}{2\sigma}\begin{bmatrix} f_{13} + f_{31} \\ f_{31} - f_{21} + f_{23} \end{bmatrix} + \mathcal{O}(\epsilon^3),\\
E_t[(\hat{c}_{23,t+1} + \tfrac{\hat{P}_{23,t+1}}{\sigma})\hat{R}_{x,t+1}] &= \tfrac{1}{2\sigma}\begin{bmatrix} f_{32} - f_{12} + f_{13} \\ f_{23} + f_{32} \end{bmatrix} + \mathcal{O}(\epsilon^3),
\end{aligned} \tag{A11}$$

where $c_{ij,t} = \frac{c_{i,t}}{c_{j,t}}$ and $P_{ij,t} = \frac{P_{i,t}}{P_{j,t}}$ denote cross-region consumption and price ratios of i to j, which constitute a vector of price-adjusted consumption differential defined as

$$\frac{\hat{cp}'_t}{\sigma} = [\hat{c}_{12,t} + \frac{\hat{P}_{12,t}}{\sigma}, \hat{c}_{13,t} + \frac{\hat{P}_{13,t}}{\sigma}, \hat{c}_{23,t} + \frac{\hat{P}_{23,t}}{\sigma}]. \tag{A12}$$

Equations A11 can therefore be re-written in the vector form as

$$E_t[\hat{cp}_t \hat{R}'_{x,t+1}] = \frac{\mathcal{F}}{2} \equiv \frac{1}{2} \begin{bmatrix} f_{31} - f_{32} + f_{12} & f_{31} - f_{21} - f_{32} \\ f_{13} + f_{31} & f_{31} - f_{21} + f_{23} \\ f_{32} - f_{12} + f_{13} & f_{23} + f_{32} \end{bmatrix} + \mathcal{O}(\epsilon^3). \tag{A13}$$

On the left hand side of this portfolio determination equation are two components: inflation-adjusted consumption differential $\hat{cp}$ and excess financial returns $\hat{R}_x$. Both components can be expressed in terms of region-specific innovations

$$\epsilon'_t = [\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t}], \tag{A14}$$

whose coefficients, as a function of portfolio choice, need to satisfy equation A13 in the equilibrium of the model. Let $\alpha_{i,j}$ represent j's holding of asset i, then the unknown portfolio matrix scaled by the discount factor β and the region's steady-state output $\bar{Y}$ to be solved in this three-economy framework is

$$\tilde{\alpha} = \frac{1}{\beta \bar{Y}} \begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} \\ \alpha_{2,1} & \alpha_{2,2} \end{bmatrix}, \tag{A15}$$

while the remaining holdings $\alpha_{3,j}$ and $\alpha_{i,3}$ can be recovered from each region's budget constraint and asset market clearing condition respectively. Given the portfolio arrangement, excess portfolio return is defined as

$$\xi_t = \tilde{\alpha}' \hat{R}_{x,t}. \tag{A16}$$

Region-specific productivity shocks ϵ_t affect the two components in equation A13 both directly and indirectly through ξ_t:

$$\hat{cp}_{t+1} = D_1 \xi_{t+1} + D_2 \epsilon_{t+1} + D_3 z_{t+1} + \mathcal{O}(\epsilon^2), \tag{A17}$$

$$\hat{R}_{x,t+1} = R_1 \xi_{t+1} + R_2 \epsilon_{t+1} + \mathcal{O}(\epsilon^2), \tag{A18}$$

where R_1, R_2, D_1, D_2, D_3 are the coefficient matrices extracted from the first-order conditions of the model. R_1 and D_1 capture the response of the two components (consumption differential and excess asset returns) to excess portfolio returns; R_2 and D_2 capture their response to productivity shocks; and D_3 are their response to other state variables in the model summarized by z. In addition, using $\xi_{t+1} = \tilde{\alpha}' \hat{R}_{x,t+1}$ allows us to express ξ_{t+1},

$\hat{cp}_{t+1}$, and $\hat{R}_{x,t+1}$ in terms of ϵ_{t+1} only:

$$\xi_{t+1} = \tilde{H}\epsilon_{t+1}, \qquad where \qquad \tilde{H} = \frac{\tilde{\alpha}' R_2}{1 - \tilde{\alpha}' R_1}; \tag{A19}$$

$$\hat{cp}_{t+1} = \tilde{D}\epsilon_{t+1} + D_3 z_{t+1} + \mathcal{O}(\epsilon^2), \qquad where \qquad \tilde{D} = D_1\tilde{H} + D_2. \tag{A20}$$

$$\hat{R}_{x,t+1} = \tilde{R}\epsilon_{t+1} + \mathcal{O}(\epsilon^2), \qquad where \qquad \tilde{R} = R_1\tilde{H} + R_2. \tag{A21}$$

Now that we have examined the two components in equation A13 separately as functions of innovations ϵ_{t+1}, we can multiply them to evaluate the portfolio determination condition:

$$E_t[\hat{cp}_t \hat{R}'_{x,t+1}] = \tilde{D}\Sigma\tilde{R}' = \frac{\mathcal{F}}{2}. \tag{A22}$$

In terms of calibration, we follow the steps below to numerically estimate bilateral financial frictions f_{ij}. First, we extract coefficient matrices R_1, R_2, D_1, D_2, and the response of the relative output differential $\hat{y}_{ij} = \hat{y}_i - \hat{y}_j$ to shocks from the first order conditions in the model. In particular, we take the first order derivative of output differential to productivity shocks

$$Dy = \frac{\partial y_{ij}}{\partial \epsilon}, \tag{A23}$$

where ϵ is the vector of productivity shocks defined in A14. We use the same method to capture the response of the relative consumption differential $\hat{c}_{ij} = \hat{c}_i - \hat{c}_j$ to shocks

$$Dc = \frac{\partial c_{ij}}{\partial \epsilon}, \tag{A24}$$

which based on equation A20 is influenced by portfolio choice $\tilde{\alpha}$ from A15 together with coefficient matrices R_1, R_2, D_1, D_2 calculated earlier. The coefficient of consumption risk sharing $\hat{\beta}_{ij}$ can therefore be approximated as

$$\hat{\beta}_{ij} = \frac{\partial c_{ij}}{\partial y_{ij}} = \frac{Dc}{Dy}. \tag{A25}$$

After we compute $\hat{\beta}_{ij}$ for each productivity shock following the steps above using the first-order dynamics of the model, we take the mean value of $\hat{\beta}_{ij}$ across shocks to get a state-pair's overall consumption risk sharing and compare it with the coefficient estimated with the method from the empirical section which serves as a targeted moment. We loop over different portfolios $\tilde{\alpha}$ until the model-predicted coefficient of risk sharing matches its empirical moment. After that, we plug the calibrated portfolio $\tilde{\alpha}$ in $\tilde{D}$ and $\tilde{R}$ of equation A22 to find matrix $\mathcal{F}$. Lastly, we recover bilateral financial frictions from this matrix of financial frictions based on equation A13.