Variable Worker Attachment, Directed Search, and Labor Market Dynamics

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15 August 2019

Abstract

This paper explores the role that two aspects of worker attachment–beliefs about finding a job and state-dependent skill change–play in the dynamic behavior of key labor market variables. I develop a model of directed search where workers lose skills when unemployed, gain them while employed, and can learn about the job finding technology by experience searching for employment. The calibrated model finds that uncertainty in job finding increases the volatility of labor market variables, while skill change decreases it. These findings are novel. In the context of skill change, these results are the opposite of what is found in models of random search with Nash bargaining because agents are unable to adjust their search behavior to limit the exposure to the costs of skill decay.

JEL classification: D83, E24, E32, J63, J64

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1 Introduction

Understanding the experiences of job seekers when looking for work is important for understanding the problems of, and choices made by, labor market participants. Insofar as these experiences influence a worker's desire to work, the present paper studies how variable worker attachment affects where individuals look for employment and, in turn, how this affects the behavior of aggregate labor market variables. In particular, this research aims to better understand how two (potentially time varying) aspects of labor market attachment–beliefs about job finding and state-dependent skill change–interact with aggregate shocks and drive labor market dynamics. This is accomplished by developing a quantitative model of directed search, calibrated to the U.S. economy, where workers can learn about the scale parameter of the matching function and lose (gain) skills when unemployed (employed).

Whereas attachment is often treated as a binary designation, the present work considers it along a continuum. In the context of the model presented below, a worker is said to be "less attached" the more her skills have depreciated or her beliefs about finding a job have deteriorated.[1] While the *extensive* distinction of active vs. inactive is an interesting one–and natural to think about in the context of variable worker attachment–the current research instead focuses on the *intensive* margin. In other words, I abstract away from the participation decision. Though this is done *not* without loss of generality, it makes the findings more comparable to the existing literature and hopes to serve as a step toward further study of this margin.

This paper makes several contributions. First, it advances theory to handle rich heterogeneity in these two aspects of variable worker attachment in a quantitatively feasible manner. Second, the model produces novel insights about how job finding beliefs and skill change affect aggregate labor market variables. Focusing on the unemployment rate, the calibrated model finds that (optimistic) beliefs lead to an increase in its mean, variance, and cyclicality (measured as the absolute value of the correlation with labor productivity). To the best of my knowledge, this is the first formal assessment of the dynamic implications of

[1] This usage is consistent with other work, for example, by Krueger et al. (2014) who find that attachment declines as unemployment tenures increase. For other examples of *similar* usage, see Abraham and Shimer (2001).

biased job-finding beliefs. Skill change, on the other hand, is found to decrease the unemployment rate, its variance, and cyclicality. Notably, these findings are the opposite of those found in Diamond-Mortensen-Pissarides (DMP) settings with random search (Ortego-Marti, 2017a; Lalé, 2018).

A general theme in these results is the distinction of *compositional* and *individual* effects. That is, how beliefs and skill change affect the types of workers searching throughout a business cycle versus how they directly affect their choices. In standard DMP settings the compositional effect is the only channel that operates. For workers, directed search allows another margin with which to adjust so as to limit, for example, exposure to the costs of skill decay. Third, because these two aspects of worker attachment generate antagonistic implications for labor market variables, I "turn off" both mechanisms in order to get an idea about which feature dominates. By correcting beliefs and eliminating skill change, I find that the belief mechanism generally dominates. Relative to this otherwise similar economy, the baseline model (i.e. with beliefs and skill change) is found to have a higher mean level of unemployment, lower variance, and higher cyclicality.

In section 2 I situate the current work among the broader literature. Most related is recent work by Spinnewijn (2015) and Mueller et al. (2018) which has found a great deal of optimistic bias in the elicited beliefs of job seekers' assessments of their job-finding probabilities. Next, a more extensive literature starting with Pissarides (1992) and Ljungqvist and Sargent (1998) has noted the importance of skill decay in explaining, for example, the negative effects of unemployment on wages and productivity. This topic is further motivated in section 3. Using Current Population Survey (CPS) data I find that roughly 50% of individuals designated as "marginally attached" cite beliefs about job finding and (or) skills as the primary reason for having not recently looked for work. Additionally, I make use of the Survey of Consumer Expectations (SCE), which has questions eliciting beliefs about individuals' job-finding probabilities, and find that unemployed workers believe that the monthly job-finding probability is twice as high as what is realized on-average.

I formulate a model in section 4, calibrate it in section 5, and then use it to understand how beliefs and skill change drive labor market dynamics in section 6. It features a frictional labor market with directed search by heterogeneously productive workers who differ also in

their skills and beliefs about the job finding probability. Here, skills control how much (i.e. what fraction) of a worker's idiosyncratic, *potential* productivity is available when producing. Skills can increase while employed, but can decrease while unemployed, capturing notions of skill decay that can affect agents' hazard out of unemployment as spell-length increases. Beliefs about finding a job, on the other hand, reflect uncertainty over the matching technology and are updated through experience searching in the labor market. In particular, agents are uncertain about the scale parameter of the matching function, and this uncertainty affects which sub-markets they search in. Optimistic workers, those who believe the job finding probability is generally higher than it is, are "picky," searching in relatively high-value and slack sub-markets. Pessimistic workers, those who believe the probability is generally low, "settle," searching in low-value and tight sub-markets. In equilibrium, those who are pessimistic exit unemployment quickly, and those who are optimistic stay unemployed longer, naturally leading to observed on-average optimism in the unemployed pool.

With directed search, vacancy-posting firms offering bilaterally efficient contracts are able to back out the worker's type, meaning that the expected value of finding a worker in a given sub-market does not depend on the endogenous distribution of workers across employment states. The induced self-selection from directed search coupled with free entry yields a meeting probability that similarly does not depend on this distribution. This property thus carries through for workers' value and policy functions, including the joint value of a match, greatly simplifying the computation of these objects since one does not have to keep track of a highly-dimensional distribution–or its evolution–begot from a great deal of individual heterogeneity. In other words, the environment admits a (unique) BRE and can thus be solved outside of the steady state, facilitating study of the dynamic implications of variable worker attachment.

Shocks to the production function affect the vacancy posting behavior of firms and is the channel through which aggregate fluctuations affect labor market variables. When aggregate productivity is high, firms post more vacancies in a given sub-market and the unemployment rate decreases. Job finding beliefs and skill change interact with this mechanism in interesting ways. The calibrated model finds that uncertainty in job finding increases the unemployment rate, its variance, and cyclicality. This result is driven by two primary forces, one of which is

compositional. Since the degree of optimism is pro-cyclical, and because optimism reduces the tightness of the sub-market job seekers search in, there is downward pressure on the variance and pro-cyclicality of the job finding probability and thus downward pressure on the variance and counter-cyclicality of the unemployment rate. The other force is idiosyncratic and works in the opposite direction as the previous. Optimism makes the perceived cost of unemployment lower, making workers more inclined to "wait it out." On the one hand, optimistic workers will respond relatively more to positive shocks by searching in tighter sub-markets. On the other, they will also search in relatively slacker sub-markets in response to negative shocks as the assessed cost of doing so is small. Overall the model finds that this latter effect dominates, and that job-finding uncertainty increases the variance and counter-cyclicality of the unemployment rate.

Skill change, in contrast, is found to decrease the variance and counter-cyclicality of the unemployment rate. Compositionally, skill loss and accrual results in pro-cyclical human capital. Since firms value higher skilled workers and post more vacancies targeting their labor, there is upward pressure on volatility measures of labor market variables. At the same time, skill change makes unemployment more costly for workers. As a result, workers search in tighter sub-markets in order to limit their exposure to skill loss, decreasing measures of volatility. Of note is that this latter motive does not exist in settings with random search and Nash bargaining (the DMP setup). There, all action from the effects of skill change comes through the vacancy posting behavior of firms and the bargaining position of workers but *not* on a margin of adjustment for these workers to choose where to apply for work. As a result, they find that skill change increases volatility measures.

The above findings ultimately reveal a tension about the role of variable worker attachment in the behavior of labor market aggregates. The upward pressure on volatility from beliefs and the downward pressure from skill change begs the question of which affect might dominate when considering an otherwise similar model without worker attachment. To make this determination, I calibrate such a model, one easily nested in the formal framework presented below, and simulate both in order to determine which effect, if any, has any bite in the aggregate. I find that, generally speaking, the qualitative implications of the optimistic beliefs channel "wins the race."

2 Related Literature

The current paper is most closely related to work with survey data indicating the ubiquity of bias in job seeker expectations about their job-finding probabilities. Spinnewijn (2015), using data from from a survey conducted by Price et al. (2004), finds that searchers expect to find a job within 6.8 weeks, but actually find jobs in about 23 weeks. In the paper's focus on unemployment benefit policy, the author advises that increasing benefits may be optimal as workers do not adequately save for (or manage their savings during) unemployment Mueller et al. (2018) explores the implication for optimistic beliefs on long-term unemployment using data from the SCE and the Survey of Unemployed Workers in New Jersey. Finding qualitatively similar on-average optimism of the searching pool, they determine that about 15% of the high incidence of long-term unemployment is explained by slow unemployment hazard induced by these beliefs. Also using SCE data, I find evidence that workers, on average, believe that the job-finding probability is roughly twice as high as what is realized at the monthly frequency. Whereas the other works focus on the provision of unemployment benefits and the disentangling of the sources of long-term unemployment, the present paper considers the dynamic implications of these findings.

Also related is work, starting with Pissarides (1992) and Ljungqvist and Sargent (1998), establishing the importance of the negatives of unemployment on labor market outcomes through the loss of skills. Differences in (the rates of) skill loss across industries, occupations, and countries have been put forth as crucial to understanding unemployment patterns and productivity measurements (Layard et al., 2005; Ljungqvist and Sargent, 2007a,b, 2008; Ortego-Marti, 2016, 2017b,c; Rogerson, 2005; Wasmer, 2006). Regarding dynamics, Ortego-Marti (2017a) and Lalé (2018) find that skill loss augments volatility of labor market variables. The latter finds that, within a DMP framework, it can account for 20-45% of the difference in labor market volatility found in data and what is produced by models. As noted earlier, the setting developed below assumes that workers have an additional margin with which to adjust, a difference I find to be important.

This work more generally relates to two broad literatures on *optimal unemployment insurance* and *unemployment duration / dependence* to the extent that worker beliefs and skill

change may affect the incentives of job seekers and thus feed into their unemployment tenure. Of concern is the provision of benefits to individuals in order to provide insurance against the costs of unemployment while maintaining their incentives to look for work. Increasing benefits induce workers to search for better jobs, leading to longer spells of unemployment. An ongoing debate asks whether these benefits should increase, decrease, or be flat over the course of an unemployment spell (Baily, 1978; Hopenhayn and Nicolini, 1997; Chetty, 2006; Shimer and Werning, 2006, 2007, 2008; Pavoni, 2009; Nekoei and Weber, 2017; Kolsrud et al., 2018).

Integral to this discussion is the duration of unemployment. From a social policy perspective, long tenures of unemployment can be very detrimental to workers, and something governments may want to ameliorate. Of note specifically is the distinction of "true" duration dependence from heterogeneity in job-finding probabilities (sometimes called "population-level" duration dependence) (Machin and Manning, 1999; Ljungqvist and Sargent, 2008; Krueger et al., 2014; Alvarez et al., 2016; Eubanks and Wiczer, 2016; Fernandez-Blanco and Preugschat, 2018). "True" duration dependence refers to the notion that the probability of finding a job is directly affected by the length of the unemployment spell, the prime example being skill decay. Measuring this in the presence of innate job-finding heterogeneity is difficult, but important for the purposes of constructing optimal and incentive-conforming policy. The treatment of worker attachment here encompasses notions of true dependence (job finding beliefs and skill change).

Similarly related is a growing literature on the *participation* margin (Garibaldi and Wasmer, 2005; Haefke and Reiter, 2006; Davis et al., 2006; Veracierto, 2008; Barnichon and Figura, 2016; Julien and Mangin, 2017; Tuzemen and Van Zandweghe, 2018). While much research, including this one, makes a two-state abstraction of labor market statuses (employment and unemployment), there is a growing interest in taking more seriously the implications of the inactive, or out of the labor force, state. Pries and Rogerson (2009) advise that not accounting for this third state may be fine in some cases, but a "serious omission" in others, for example when trying to understand cross-county labor market patterns. Dynamically, Elsby et al. (2015) find that a third of the cyclical behavior in the unemployment rate is driven by this extensive labor supply margin. Naturally, the concept of worker attach-

ment is closely related to this topic. Indeed the marginally attached designation by the BLS noted earlier is excluded in the often-cited U3 measure of unemployment. Again, though the inclusion of a discrete participation decision is not made here, I view the continuous implementation of variable worker attachment as an initial step toward tackling the issues and implications of inactivity.

Theoretically relevant is work by Menzio and Shi (2010a,b, 2011) on the concept of Block Recursive Equilibria (BRE). In dynamic settings with a lot of idiosyncratic heterogeneity, some of which is unobservable to firms, solving models numerically can quickly become infeasible as agents must keep track of a high-dimensional and endogenous distribution of searching worker types (a "curse of dimensionality"). This equilibrium concept is such that the agents' value and policy functions depend on the aggregate state only through realizations of the aggregate shock (and not on the "cursed" distribution of workers across employment states). The cited work is influential in establishing the theory of BRE for a wide range of applications, most notably for environments with search on-the-job. Essential for the present work is that these equilibria can be solved outside of the steady-state, supporting examination of the model's dynamic behavior. Novel to the environment presented below is that, though there is no search on-the-job, subjective beliefs about the matching function introduce a potential violation of the block recursive structure. Under reasonable conditions, however, these "technically" unobserved beliefs can be backed out so that firms need not track the distribution of priors across searching workers.

3 Data

3.1 The Components of Labor Market Attachment

Typical definitions of labor market attachment, for instance those used by state governments when assessing the provision of unemployment benefits, are categorical: one either *is* or *is not* considered to be making reasonable efforts to find gainful employment. Because these efforts are often unobserved by economists, the term sometimes describes those demographic groups most likely to participate in the labor market and is more synonymous with "desire

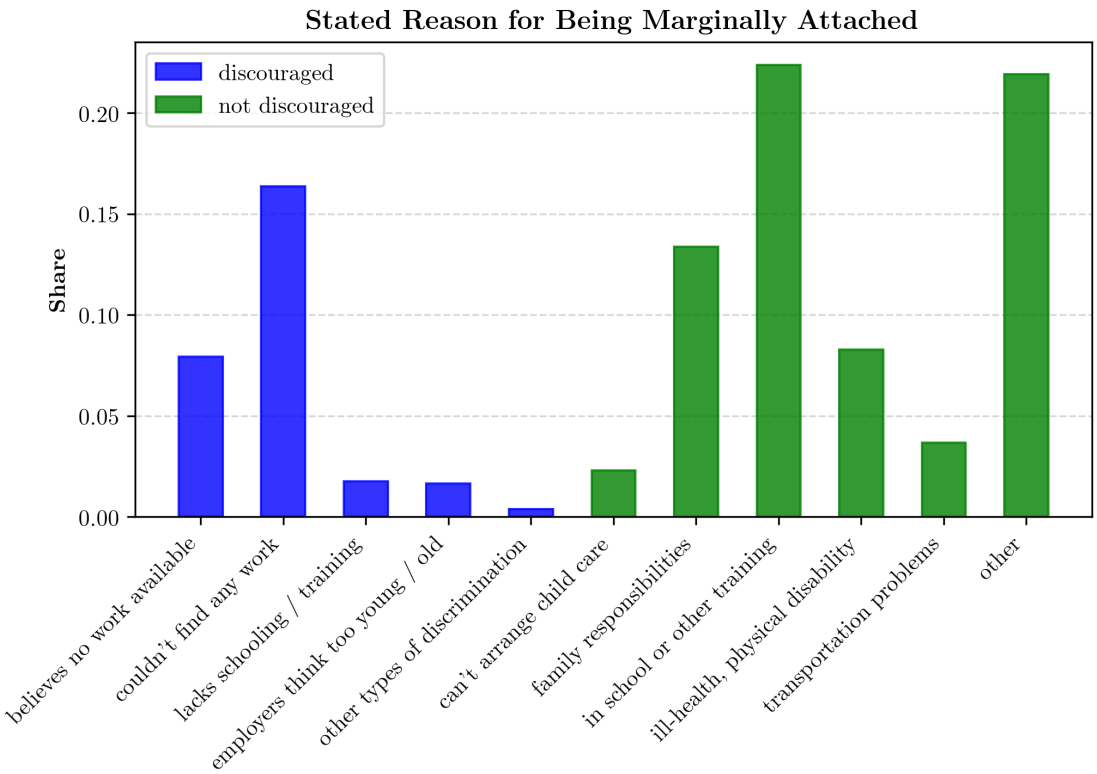

Figure 1: The above displays the share of the marginally attached population by the reason given for not searching for work. The BLS classifies an individual as marginally attached if the respondent cites any of the above reasons for not searching for work in the previous 4 weeks *and* has looked for work at least once in the previous 12 months. These shares are monthly averages calculated using the CPS basic monthly files from January 1994 through December 2018.

to work." The Bureau of Labor Statistics (BLS) identifies a similar, yet distinct, group in those who are *marginally attached* to the labor force. These individuals would work, were it available, but were not counted as unemployed for one of several reasons. In addition to having looked for work at least once in the previous 12 months, an individual is considered marginally attached if they gave any of the reasons presented in Figure 1 for not looking for work in the previous 4 weeks. The further distinction of *discouraged* workers is sometimes used, and references recent negative experiences of individuals looking for work. In any regard, roughly 50% of marginally attached workers cite beliefs about job search and (or) skills as the reason why they have stopped looking for work. The remaining respondents largely cite discrimination, family, health, and transportation issues as the reason for not searching.

I restrict the focus of this paper to beliefs about job finding in part because they comprise a large proportion of these motives given for the BLS classification of marginally attached. Further, noting the overlap in some of the stated reasons, they are likely interrelated. Further, both conceivably vary at the business cycle frequency and so plausibly play an interesting role in the cyclical behavior of labor market variables. To the extent that other aspects of this BLS measure of attachment also varies at this frequency, the fraction of respondents citing them is relatively small or the mechanisms underlying them are beyond the scope of the structural modeling presented here (e.g. family responsibilities or "other" reasons).

3.2 Bias in Job-Finding Beliefs

Having shown that beliefs about one's labor market prospects are an important component of an individual's attachment, I next turn to more concretely characterize this bias in terms of job-finding probabilities. To do so I make use of the SCE, a monthly survey with a representative sample of household heads in the US. Run by the Federal Reserve Bank of New York, the survey started in December 2012 and elicits the perceptions of unemployed workers' job-finding prospects. Its rotating panel structure allows for ex-ante beliefs to be linked to actual, realized outcomes. Specifically, the survey asks unemployed individuals the following.

> *And looking at the more immediate future, what do you think is the percent chance that within the coming 3 months, you will find a job that you will accept, considering the pay and type of work?*

To get an understanding about the bias of job seekers, I follow a procedure similar to that in Mueller et al. (2018). Because the question is asked about the coming 3 months, the sample is restricted to those observations which are followed with at least 3 consecutive responses (that is a total of 4 consecutive months responding to the survey). This is in order to verify whether or not these respondents did ultimately find jobs. Next, because the panel is somewhat limited in how long individuals are tracked (that is we do not consistently observe multiple unemployment spells for each respondent), I calculate the realized rate for respondents in period t as the fraction of unemployed individuals in that period who found

Bias in Job-Finding Beliefs

Horizon	Believed Prob.	Realized Prob.	Difference	Ratio	Observations
3-month	0.47832	0.3680	0.1141	1.4958	871
	(0.0137)	(0.0010)	(0.0142)	(0.0529)	
1-month	0.2696	0.1459	0.1261	2.1373	871
	(0.0127)	(0.0005)	(0.0125)	(0.0978)	

Table 1: The above gives the means of the believed probabilities and realized probabilities of finding a job in the SCE for both the 3-month horizon and 1-month (imputed) horizon. The sample is restricted to respondents aged 25-65 and with at least 3 follow-up interviews. standard errors are reported in parentheses. Survey weights are used in all calculations.

jobs in $t+1$, $t+2$, or $t+3$. That is, though nothing concrete can be said about individual bias, this exercise allows the examination of the average bias in expected job finding probabilities.

Next, because I later calibrate to a monthly frequency, I impute a rough estimate of a 1-month probability as follows. First, I assume that the arrival of jobs follows a Poisson process and that they are believed to arrive at some continuous rate $\hat{\lambda}$. As in Shimer (2012), I further assume that an econometrician only observes the fruits of job search at discrete points $t = 1, 2, 3, \ldots$, where $[t, t+1)$ is "period t." The subjective probability that a job will arrive by t follows an exponential distribution: $\Pr(t; \hat{\lambda}) = 1 - e^{-\hat{\lambda} t}$. I use this to get an estimate for a one-month probability as follows. For each individual with an elicited 3-month probability, calculate their implied $\hat{\lambda}$.

$$\hat{\lambda} = -\frac{1}{3} \ln\big(1 - \Pr(3)\big) \tag{1}$$

Then, I use this to evaluate $\Pr(1; \hat{\lambda})$. The same procedure is used in order to back-out the realized one-month probability. Last, I condition on workers aged 25-65. The results of this exercize are displayed in Table 1. On average, workers believe the monthly job finding probability is about 12.6 percentage points higher than what is realized. This translates to a belief that the job-finding probability is 2.14 times higher than what is observed in the sample. To give an idea about the distribution of beliefs, I plot the histogram of elicited beliefs relative to the average realized probability of finding a job in Figure 2. Further, I include a line indicating the "correct" value; that is, the value that would arise if everyone's

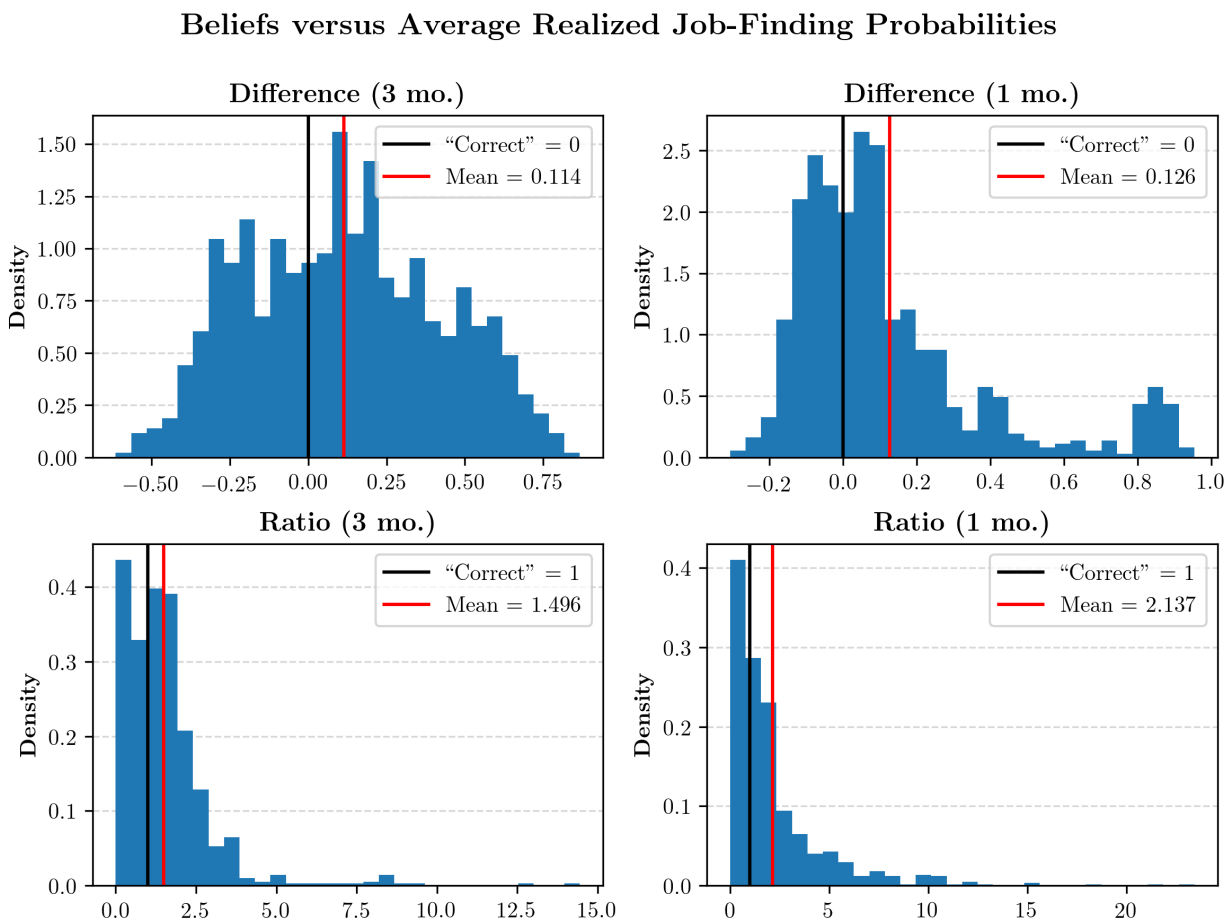

Figure 2: The above displays different measures of bias in job-finding expectations in the SCE sample. Both for the 3-month horizon and 1-month (imputed) horizon and both as a difference and a ratio, the distribution of elicited beliefs about the probability of finding a job is plotted relative to the average realized probability of finding a job for the month the respondent was surveyed. The implied "correct" value is plotted in black. That is, the value of the measure if everyone's beliefs exactly equaled the realized probability.

beliefs were in-line with realizations. All mean probabilities and differences are statistically distinguishible from 0 and the ratios are both statistically distinquishible from 1, suggesting bias in job-seekers expectations of finding jobs.

4 Model

In this section I formulate a model that I later use to understand how beliefs and skill change drive labor market dynamics. Key to its construction is the ability to handle rich heterogeneity in these two aspects of variable worker attachment in a quantitatively feasible

manner.

4.1 Environment

A unit measure of workers and positive measure of firms populate the economy in discrete time, discounting the future with a factor $\widetilde{\beta} \in (0,1)$. Workers live indefinitely, dying (with replacement) at the end of a period with probability $\delta \in (0,1)$. They can be either employed and producing output (E) or unemployed and directing their search for work into sub-markets (U) Aside from their employment status, workers are heterogeneous in three respects: their individual productivity, skill-level, and beliefs about the job finding process.

First, individuals have innate, *potential* productivity differences at work, $z \in \mathbb{R}_+$, which are drawn at the start of life from G_z and never change. Access to this productivity is governed by skills, $h \in H = \{h_1, h_2, \ldots, h_N\}$, $h_r \in (0,1)$ and $h_{r+1} > h_r$ for $r = 1, \ldots, N$, which affinely affect what fraction of their innate productivity is realized when producing at work. That is, *actual* idiosyncratic productivity is given by zh. These skills are drawn initially from some discrete distribution G_h at the start of life but can change over time depending on the agent's employment state. In particular, let $\pi_i(h'|h)$ be the transition probability matrix for workers in employment state $i \in \{E, U\}$, where *primes* denote variables next period.

Finally, workers differ in their beliefs about the job finding process. It is assumed that workers are uncertain about the scale parameter of the matching function, $\mu \in \mathbb{R}_+$, and that they can learn about it through experience searching in the labor market. For the time being, I let $p(\mu)$ denote an arbitrary function that defines a worker's prior beliefs about μ.[2] Work by Spinnewijn (2015) and Mueller et al. (2018) motivates modeling uncertainty in this way in their finding that individuals' beliefs about the probability that they find work in the following month are (optimistically) biased on average and that there is a great deal of dispersion in the cross section. To the extent that workers differ in this way, choose where to apply for work, and because I follow the literature in modeling job finding frictions with a CRS matching function, uncertainty over the scale parameter (as opposed to the elasticity of the job finding probability with respect to the vacancy-to-applicant ratio) is natural.

Taken all together, as skills and beliefs adjust over the course of an unemployment episode,

[2] More on beliefs is laid out below.

workers re-direct their search to, usually, tighter sub-markets. For now, it is assumed that only z and h are observable to employers.

In the spirit of Menzio and Shi (2011), contracts between workers and firms are assumed to be bilaterally efficient: they maximize the joint surplus of a match. Briefly, Menzio and Shi (2010a) show under fairly general assumptions that profit maximizing contracts are bilaterally efficient if the contract space is complete–an assumption I, in turn, make. The output of an employed worker is given by Azh, where $A \in \mathbb{R}_+$ is common to all firms and time varying. Unemployed workers receive unemployment utility $b \in \mathbb{R}_+$ and direct their search to sub-markets indexed by x and worker observables. $x \in \mathbb{R}_+$ denotes a firm's promised present expected value of employment, delivered to workers by way of choices for some wage and a separation probability $s \in [\underline{s}, 1)$.[3] In order to find workers, firms post vacancies in these sub-markets at a cost $\kappa \in \mathbb{R}_+$ per period. The aggregate state of the economy at the beginning of any period is given by $\psi \in \Psi = (A, u)$. The last element in ψ is a function that maps a worker's type into the measure that are unemployed and searching for work.

Hiring in a given sub-market takes place according to a CRS matching function that takes the measures of vacancies and unemployed workers in the sub-market as inputs. Ignoring sub-market indexing for the time being, $m = \mu v^{\eta} u^{1-\eta}$, where v is the measure of vacancies. The probability of finding a job for a worker is given by $f(\theta) \equiv m/u = \mu\theta^{\eta}$, where $\theta \equiv v/u$ is the standard definition labor market tightness in a particular sub-market. Similarly, from the firm's perspective, the probability of filling a vacancy is given by $q(\theta) \equiv m/v = f(\theta)/\theta$. What is important for decisions, however, is what the worker *believes* her job finding prospects are. A worker has beliefs of finding a job $\hat{f}(\theta)$ induced by the prior $p(\mu)$:

$$\hat{f}(\theta) = \int f(\theta; \mu) p(\mu) d\mu, \tag{2}$$

where notation of the sort $f(\theta; \mu)$ is sometimes used to stress the reliance of the job finding probability on μ. The *job finding* beliefs for searching workers, i.e. the beliefs about the function $f(\cdot)$, are updated according to Bayes' Rule at the end of each period. Specific details on how learning takes place, and the results that support it, are outlined below.

[3] $\underline{s}$ designates the exogenous probability that a match is dissolved.

LEMMA 1. The prior and posterior means of μ are sufficient statistics of the belief structure for the worker's search decision.

Proof. See Appendix 8.1. □

The above result, in other words, advises that no moments other than the prior and posterior means of μ are necessary for assessing the worker's search choice. Noting the practical, i.e. computational, limitations associated with an arbitrary prior distribution in the worker's state, I assume that the priors follow a conjugate structure so that these sufficient statistics can be characterized by a finite-dimensional object.

LEMMA 2. Let $y \equiv \mathbb{1}(\text{find job})$ denote the indicator function for whether a searcher finds a job. Next, suppose that there is an initial belief about the mean of μ, $\hat{\mu}$, that induces a choice to search in a sub-market with tightness θ, producing an expected job finding probability $\hat{f} = \hat{\mu}\theta^{\eta}$. Then

(i) there exists a beta distribution with (hyper-) parameters α and γ that can express the prior belief of the job finding probability such that the prior mean is $\hat{f}$ and

(ii) the associated posterior is beta distributed with parameters $\alpha' = \alpha + y$ and $\gamma' = \gamma + 1 - y$,

(iii) the posterior mean of μ is

$$\hat{\mu}' = \left(\frac{\alpha'}{\alpha' + \gamma'}\right)\theta^{-\eta}.$$

Proof. See Appendix 8.2. □

The structure laid out in the above two results admits a rich belief and learning mechanism for searching workers that can be characterized by a finite number of state variables. This structure, however, is perhaps simultaneously *too* flexible and *too* restrictive. Regarding flexibility, given some $\hat{\mu}$ that induces a choice to search in a sub-market with tightness θ, there is an infinitely large set of pairs (α, γ) that can achieve the prior mean $\hat{f}$. That is, there is an issue of initiating α and γ for an individual worker. Regarding restrictiveness, I note that α and γ can roughly be interpreted as the number of "successes" and "failures" of job search, respectively. In an environment where workers are (re-) directing their search

over time, it is unrealistic that the series of chosen sub-markets have exactly a subjective job-finding probability of $\alpha/(\alpha+\gamma)$, incremented appropriately.

To address both of these issues, I make the following simplification: workers only keep track of the prior belief $\hat{\mu}$ and the total number of periods they've searched in the labor market. The process works as follows. $\hat{\mu}$ is drawn from some distribution $G_{\hat{\mu}}$ at the start of life and start out with an initial $\alpha+\gamma=\alpha_0+\gamma_0$ which is fixed. This induces search in some sub-market with tightness θ yielding a subjective job-finding probability $\hat{f}$. Both α_0 and γ_0 can then be uniquely backed-out to be consistent with $\hat{f}$ and used, in turn, to evaluate how $\hat{\mu}'$ is to be updated depending on the result of job search. For a worker, $\alpha+\gamma-\alpha_0-\gamma_0$ is her job search experience and affects the *speed* $\hat{\mu}$ is updated.[4] Further, interpreting $\alpha+\gamma$ in this way, I assume that it is observable by employers and, along with z and h, make up an individual's "résumé." In an abuse of notation, I redefine $p \in P = (\hat{\mu}, \alpha+\gamma)$ to denote the *prior* belief variables, while $p_y \in P_y = (\hat{\mu}', \alpha'+\gamma')$, $y \in \{0,1\}$, will designate the *posterior* beliefs for search outcome y.

In order to visualize how this updating works in practice, I plot an example in Figure 3. That is, I plot the key state and choice variables for a worker as she looks for a job. So as to highlight just the updating mechanism, the example assumes that a worker's skills and the aggregate state are constant. The particular individual is tracked for 10 periods, starting in period $t=0$. She starts out with 18 periods of search experience and initially believes that the scale parameter of the matching function is relatively high at 3.1. Since she believes job-finding is easy, she chooses to search in relatively slack sub-markets, and thus has a relatively low probability of actually finding a job. As she searches and fails to find a job, her beliefs deteriorate, and she responds by searching in tighter sub-markets, raising the likelihood she finds a job. At $t=6$ she successfully finds a job; her beliefs are adjusted upward slightly, and the probability she finds a job thereafter drops to 0 as there is no on-the-job search. Note also that her search experience stops incrementing at the time she finds a job.

Each period is characterized by five stages. In the first stage, learning and skill change takes place. New realizations of h are drawn and beliefs p are updated based on the results of the previous period's search experiences y. In the second stage the aggregate state ψ

[4] If $\alpha+\gamma$ is large, updates are small. If the sum is small, updates are large.

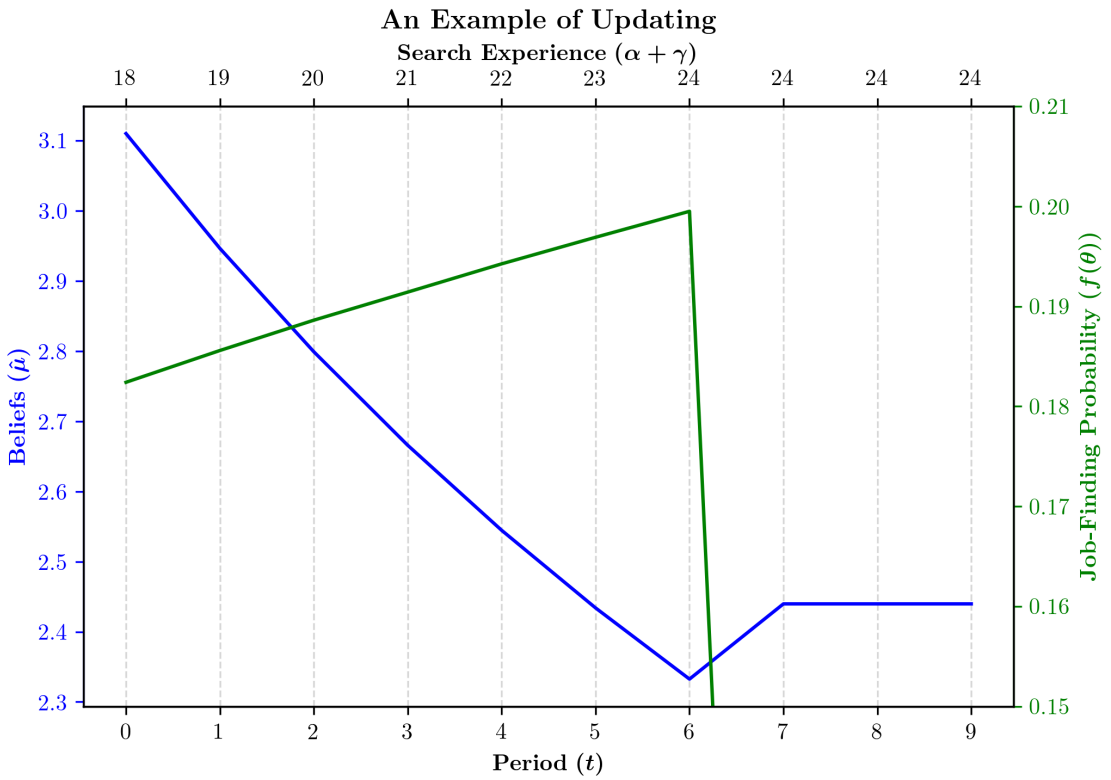

Figure 3: The above graphically shows how updating works in the model. This particular individual is tracked starting in period $t = 0$ with beliefs $\hat{\mu} = 3.1$ and labor market experience $\alpha + \gamma = 18$. This individual is optimistic, choosing to search in a slack sub-market, and so has a low probability of finding a job. As unemployment tenure extends, search experience accumulates, and her beliefs are adjusted downward. As this happens, she searches in tighter sub-markets, increasing her job-finding probability. In period $t = 6$ the worker finds a job; her beliefs are revised upward as a result and her job-finding probability drops thereafter to 0 (there is no on-the-job search).

is established with a draw for A. Additionally, separations and death (with replacement) occur. Those who lose their job at this stage cannot find a job until the following period. Production and consumption occur in the third stage. Unemployed workers receive b in utility while employed workers produce Azh and are paid according to their employment contract. Vacancy posting and search happens in the fourth. Finally, in the fifth stage, matching occurs.

4.2 Value Functions

Let the market tightness function for the sub-market promising x to individuals with résumé $\phi = (z, h, \alpha+\gamma)$[5] with aggregate state ψ be written as $\theta_{x,\phi,\psi} \equiv \theta(x, \phi, \psi)$. At the start of the production and consumption stage, an unemployed worker receives b in the current period and chooses which sub-market to search in given z, h, and p. Should they find a job, which is believed to occur with probability $\hat{f}$, they earn x in expected lifetime utility, otherwise they remain non-employed next period with updated beliefs p_0.

$$\mathcal{V}_U(z, h, p, \psi) = b + \beta \mathbb{E}_{\psi'|\psi} \max_x \Big\{ \hat{f}(\theta_{x,\phi,\psi})x + \big(1 - \hat{f}(\theta_{x,\phi,\psi})\big) \sum_r \mathcal{V}_U(z, h'_r, p_0, \psi')\pi_u(h'_r|h) \Big\}, \tag{3}$$

where $\beta \equiv \widetilde{\beta}(1-\delta)$ is the effective discount factor. The joint surplus of a job includes the sum of the worker's continuation utility and the firm's continuation profit.

$$\begin{aligned} \mathcal{V}_E(z, h, p, \psi) = Azh + \beta \mathbb{E}_{\psi'|\psi} \max_s \Big\{ & s \sum_r \mathcal{V}_U(z, h'_r, p, \psi')\pi_e(h'_r|h) \\ & + (1-s) \sum_r \mathcal{V}_E(z, h'_r, p, \psi')\pi_e(h'_r|h) \Big\} \end{aligned} \tag{4}$$

4.3 Equilibrium

Due to free entry, any sub-market with a strictly positive measure of searchers will have a market tightness function that satisfies

$$\kappa \geq q(\theta_{x,\phi,\psi}) \Big[\mathbb{E}_{\psi'|\psi} \sum_r \mathcal{V}_E(z, h'_r, p_1, \psi')\pi_u(h'_r|h) - x \Big] \tag{5}$$

and $\theta_{x,\phi,\psi} \geq 0$ with complementary slackness.

Regarding the above expression, it is important to note that while sub-markets are indexed by (x, ϕ, ψ), the condition pinning down the labor market tightness is also a function of $\hat{\mu}$. At first glance, it would seem necessary for firms to keep track of the distribution of

[5] This notation is used only for brevity of exposition of the market tightness function.

prior mean beliefs of μ across searching agents by observable characteristics. If, however, there is a unique x chosen by searchers given ϕ and ψ, then firms offering x to workers in the (x, ϕ, ψ) sub-market will be able to back out $\hat{\mu}$. That is x and $\hat{\mu}$ are pegged to one another. The following proposition formalizes this idea.

PROPOSITION 1. Firms do not need to track the distribution of prior mean beliefs about μ among searching agents when evaluating their decision to post a vacancy in a sub-market.

Proof. See Appendix 8.3. □

With all of the above laid out, we can move forward to define and establish a (unique) Block Recursive Equilibrium in this environment.

DEFINITION 1. A Block Recursive Equilibrium (BRE) consists of a market tightness function θ, value functions $\mathcal{V}_U$ and $\mathcal{V}_E$, and policy functions x and s such that the following hold:

(i) θ, $\mathcal{V}_U$, $\mathcal{V}_E$, x, and s depend on ψ only through A;

(ii) θ satisfies Equation 5;

(iii) $\mathcal{V}_U$, and x satisfy Equation 3;

(iv) $\mathcal{V}_E$ and s satisfy Equation 4.

THEOREM 1. The unique recursive equilibrium is a BRE.

Proof. See Appendix 8.4. □

5 Calibration

In this section I calibrate the parameters of the model presented above using data on worker transitions, job finding beliefs, and re-employment prospects. I organize these parameters into 4 groups: those related to preferences, search and matching, aggregate productivity, and a worker's résumé. To start, a model period is chosen to be 1 month. In order to

make data and model generated series comparable I measure them as log deviations from a Hodrick-Prescott filtered trend using smoothing parameter set to 14400 where appropriate (e.g. comparing second-order moments).

Preferences are described by a discount factor $\widetilde{\beta}$, an exogenous death probability δ, and unemployment utility b. $\widetilde{\beta}$ is set to 0.9957 implying a 5% annual discount rate. δ is set to 0.0021 so that the average worker is in the labor market for 40 years (i.e. from 25 to 65). I interpret b as the value of leisure and calibrate it such that its ratio with the average labor productivity is 0.71 as in Hall and Milgrom (2008).

Search and matching frictions are described by a matching function scale parameter μ, a curvature parameter η, an exogenous separation probability $\underline{s}$, and a per-period vacancy posting cost κ. μ is normalized to 1.0 allowing interpretations of $\hat{\mu}$ to be easily made in percentage terms. For example, $\hat{\mu} = 1.2$ means that a worker's subjective job-finding probability is 20% higher than the actual probability. η is set so that the elasticity of the job-finding probability with respect to the sub-market tightness in the model is the same as in the data.[6] The model's aggregate unemployment and vacancy measures are given by $\int u$ and $\int \theta u$, respectively. The equivalent series in the data are taken to be the civilian noninstitutionalized unemployment level from the CPS and total unfilled job vacancies ("job openings") from the BLS's Job Openings and Labor Turnover Survey (JOLTS) for the period from December 2000 to December 2018.

For the job-finding probability I calculate and use the "UE" transition probability directly from the CPS basic monthly files by matching individuals in consecutive months as in Shimer (2012).[7] Since there is no on-the-job search (that is, all search comes from unemployment), η can be set directly and is estimated using a log-linear specification with linear and quadratic trends. The exogenous separation probability and vacancy posting cost, $\underline{s}$ and κ, are calibrated to target the mean "EU" and "UE" transition probabilities observed in the data (again, measured as in Shimer (2012)). The model equivalents are $\int s$ and $\int f(\theta)$.

Aggregate productivity is assumed to follow an AR(1).

[6]Note here that, because uncertainty in job finding is over the scale parameter and I assume workers observe the vacancy posting behavior of firms, it doesn't matter if one uses the subjective or actual job finding probability in the model.

[7]Note also that because I abstract away from a participation margin, I "correct" all transition probabilities accordingly. That is, so the probabilities appropriately sum to unity.

$$A' = (1-\rho)\overline{A} + \rho A + \varepsilon \tag{6}$$

$\overline{A}$ is normalized to 1.0 and $\varepsilon \sim N(0, \sigma)$. σ and ρ are chosen such that the model's implied standard deviation and autocorrelation of aggregate labor productivity is the same as in the data. Model labor productivity is given by $\int \frac{Azh}{1-u}$. In the data I use the BLS's estimate of the real output per hour in the nonfarm business sector.

I assume that the skill ladder has $N = 11$ equally spaced rungs from $h_1 = 0.5$ to $h_N = 1.0$ so that someone at the bottom of the ladder is half as productive as someone at the top for the same level of potential productivity. The associated transition probability matrices are assumed to have a structure similar to Ljungqvist and Sargent (1998): skills may only increase while employed and may only decrease while unemployed. In particular, it is assumed that the probability of increasing (decreasing) one's skill is constant across the entire ladder and equal to π_e (π_u). With the complement of these probabilities, the worker remains on the same skill rung. Further, I assume symmetry, that is $\pi_e = \pi_u = \pi$.

I calibrate these skill-transition probabilities using estimates from the literature on the returns to human capital. In particular, Ortego-Marti (2016) uses a Mincerian-style regression with data from the Panel Study of Income Dynamics (PSID) to construct unemployment histories for workers and finds that an additional month of unemployment is associated with a 1.22% drop in wages. Because the reduced-form moment, interpreted in the context of the present model, corresponds to effects from skill loss *and* belief deterioration, I target this moment in the model so that the expected loss in wages of one more month of unemployment is 1.22%. Recall, however, that workers are directing their search to continuation values and not wages. Indeed, there are many possible contingencies for w that achieve a value of x given s. For example, the wage payments might be constant, increasing, or decreasing throughout the employment tenure of a worker. For the purposes of the theory laid out above, it was unnecessary to specify precisely how wages were determined. In order "back out" wages for the target, though, it is necessary to describe a wage setting mechanism. For simplicity I assume that wages are constant over the course of an employment spell (see Appendix 8.5 for details). The expected percentage loss in wages for an individual searching worker is thus

$$\ln w(z,h,p,\psi) - \ln \mathbb{E}_{\psi'|\psi} \sum_r w(z,h',p_0,\psi')\pi_u(h'_r|h). \tag{7}$$

I use simulated data from the model to calculate the corresponding aggregate series: $\int \ln w - \int \ln w'$. Initial draws of h are assumed to come from a discrete uniform over H. All together, skills are characterized by 4 parameters and a distribution: N, h_1, h_N, π, G_h.

Last, beliefs are characterized by an initial auxillary parameter $\alpha_0 + \gamma_0$ and a distribution $G_{\hat{\mu}}$. The initial, prior mean belief of $\hat{\mu}$ is assumed to be drawn point-mass at $\hat{\mu}_0$ and, with $\alpha_0 + \gamma_0$, is calibrated such that the average belief-to-realized job-finding probability ratio and dispersion in beliefs match the SCE sample described earlier. While the mean of this ratio can be directly calculated as the average $\hat{\mu}$ of the searching pool in the simulated data, the standard deviation of the bias variable in the data are *not* equivalent to the standard deviation of $\hat{\mu}$. To ensure that model and actual data are appropriately compared, the equivalent series in the model is calculated and used. That is, the bias in the model is calculated as the ratio of $\hat{f}$ with the average *actual* job-finding probability of unemployed searchers in a given period.

In total, there are 16 parameters. 8 are set before the main calibration ($\widetilde{\beta}$, δ, μ, η, $\overline{A}$, N, h_1, h_N) and 8 are jointly determined with 8 targeted moments (b, $\underline{s}$, κ, σ, ρ, π, $\hat{\mu}_0$). Results of the calibration are displayed in Table 2.

6 Results

In this section, I use the model presented above to study the role that worker attachment plays in the behavior of aggregate labor market variables. Due to the nature of the model, analytic, comparative static exercises are limited. Thus, I use the calibrated model as a laboratory to isolate the effects that beliefs and skill change have on aggregate variables, and how aggregate shocks to productivity interact with these channels. This is accomplished in several steps.

First, I study the effects that beliefs and skill change have on the levels (means) of key model variables. This is done as follows. The economy's steady state is simulated at the

Calibration Results

Parameter	Value	Description	Target
Preferences			
$\widetilde{\beta}$	0.9957	discount factor	5% annual discount rate
δ	0.0021	death prob.	average lifespan of 40 years
b	0.71	unemployment utility	leisure-to-labor prod. ($\star$)
Search & Matching			
μ	1.0	matching function, scale	normalization
η	0.36	matching function, curvature	estimated
$\underline{s}$	0.0141	exogenous separation prob.	mean EU prob. ($\star$)
κ	3.79	vacancy posting cost	mean UE prob. ($\star$)
Aggregate Productivity			
$\overline{A}$	1.0	mean aggregate prod.	normalization
ρ	0.7431	autocorr. of aggregate prod.	autocorr. of US labor prod. ($\star$)
σ	0.0076	st. dev. of aggregate prod.	st. dev. of US labor prod. ($\star$)
Résumé			
N	11	number of skill rungs	chosen
h_1	0.5	minimum skill level	$\frac{1}{2}$ of h_N
h_N	1.0	maximum skill level	normalization
π	0.19	skill change prob.	1.22% drop in wages ($\star$)
$\hat{\mu}_0$	4.06	initial belief	mean bias ($\star$)
$\alpha_0 + \gamma_0$	58.77	initial "search experience"	st. dev. of bias ($\star$)

Moment	Data	Model
value of leisure to average labor productivity ratio	0.7100	0.7124
average job-finding probability	0.0141	0.0143
average job-separation probability	0.2608	0.2695
autocorrelation of aggregate labor productivity	0.7244	0.7230
standard deviation of aggregate labor productivity	0.0100	0.0107
average loss in wages from unsuccessful search	0.0122	0.0122
average job-finding bias	2.1373	2.1309
standard deviation of job-finding bias	0.3011	0.3175

Table 2: Results of the baseline calibration. The top panel displays the parameters and the bottom reports the moments targeted in the joint exercise. Jointly calibrated parameters are "starred" in the top panel.

mean of aggregate productivity, $\overline{A}$, where the distribution of workers across idiosyncratic states is given by the ergodic distribution associated with this level of productivity. I then separately increase the degree optimism and rate of skill change in the model and compare the new steady state values with the baseline model. Foreshadowing interest in the dynamic implications of these mechanisms, I also calculate the steady state version of the model when mean aggregate productivity is increased (by 1%). This is done in order to generate relationships between aggregate productivity and the other model variables, which will be important in understanding the model with aggregate shocks.

Next, I study how beliefs and skill change affect the dynamics of these variables. To do so, I start the various economies–the baseline model, the increased optimism model, and the increased skill change model–in the steady state and then "turn on" aggregate shocks. I then use these simulations to compare standard deviations, correlations with labor productivity, and serial correlations. Since beliefs and skill decay generate opposite implications for the dynamics of labor market variables, I calibrate an otherwise similar model with both mechanisms "turned off." That is, I correct beliefs, eliminate skill change, and simulate this economy to compare with the baseline model.

6.1 Levels

The qualitative results of the steady-state levels exercise are displayed in Table 3 and are reported relative to the baseline model. First, consider the increased optimism exercise. Here, workers are generally more optimistic, leading to a mechanical increase in $\hat{\mu}$. As a result, they are more picky, search in appropriately slacker sub-markets, decreasing the average job finding probability and increasing the unemployment rate. Since job finding happens more slowly, more time is spent unemployed, and human capital is lower. Next, consider the "increased skill change" exercise. Here, workers are more affected by skill decay when unemployed, decreasing average human capital.[8] Additionally, since unemployment is relatively more costly, workers choose to search for worse jobs (i.e. in tighter sub-markets),

[8]There is also more skill accrual when employed. However, since skills are primarily concentrated near the top of the ladder–that is the distance from the max skill level is the degree of skill decay–the increase in skill decay is what has bite in this experiment.

Qualitative Results: Levels

	h	$\hat{\mu}$	θ	f	u
↑ Optimism	−	+	−	−	+
↑ Skill Change	−	+	+	+	−
↑ Productivity	+	+	+	+	−
CB & NS	+	−	+	+	−

Table 3: The above table displays the qualitative results, relative to the baseline model, for the means of key model variables of the three comparative static exercises described above.

increasing the job finding probability, beliefs about μ, and lowering unemployment.

The last row of Table 3 presents the results when aggregate productivity is increased. Since the source of volatility comes from shocks to A, these relationships will be key in understanding some future results pertaining to the dynamics of some labor market variables. In any regard, higher productivity leads to firms posting more vacancies, increasing market tightness, job finding probabilities, and $\hat{\mu}$. Correspondingly, h is higher as less time is spent unemployed.

6.2 Dynamics

The qualitative results of the dynamics exercises are given in Table 4. Very briefly, we can see that optimism has a positive effect on volatility, cyclicality, and serial correlation, while skill change has a negative effect. Since this model is in general equilibrium, it will be useful to think of two distinct dichotomies in order to make sense of these results. The first division concerns a *composition* versus *individual* effect. That is, how do beliefs and skill change affect the composition of the unemployed pool versus the choices of individuals? The second division further separates the individual effect into *supply* and *demand* effects. In other words, how do these forces affect the choices of workers and firms?

I guide the comparisons of these dichotomies using the equilibrium (sub-) market tightness function since it embodies the decisions of both searching workers and hiring firms. It will be beneficial to contextualize how aggregate shocks affect where both the *average* worker and an individual worker searches for work. This is done by inspecting how the total derivative of the sub-market tightness function, with respect to changes in A, depends on beliefs and skill change.

Qualitative Results: Dynamics

	St. Dev.			Corr(·, LP)^2			Serial Corr.		
	θ	f	u	θ	f	u	θ	f	u
↑ Optimism	+	+	+	+	+	+	+	+	+
↑ Skill Change	−	−	−	−	−	−	−	−	−
CB & NSC	−	−	+	−	−	−	+	+	−

Table 4: The above table displays the qualitative results for the differences in the dynamics of key model variables of the increased optimism, increased skill change, and correct beliefs / no skill change economies.

To start, θ is a function of the promised payment x, résumé ϕ, and aggregate state ψ. We can use the model to rewrite this function as follows. Because the equilibrium is block recursive, ψ can be reduced to A as it does not depend on the endogenous distribution of workers across employment states. Next, given the results in PROPOSITION 1, we can replace x with $\hat{\mu}$ as firms can back out $\hat{\mu}$ from the choice of x with resume ϕ. Finally, by eliminating the résumé notation, we can write the equilibrium market tightness function as $\theta(z, h, p, A)$. Its total derivative is given by

$$\begin{aligned} d\log(\theta) = & \underbrace{\frac{\partial \log(\theta)}{\partial z} dz + \frac{\partial \log(\theta)}{\partial h} dh}_{\text{human capital}} \\ & + \underbrace{\frac{\partial \log(\theta)}{\partial \hat{\mu}} d\hat{\mu} + \frac{\partial \log(\theta)}{\partial \alpha} d\alpha + \frac{\partial \log(\theta)}{\partial \gamma} d\gamma}_{\text{beliefs}} \\ & + \underbrace{\frac{\partial \log(\theta)}{\partial A} dA}_{\text{agg. prod.}}, \end{aligned} \tag{8}$$

where logs are taken so that interpretations can be made in percentage terms. By dividing through by dA we can get an expression for the percentage response in θ from shocks to A.

$$\frac{d\log(\theta)}{dA} \approx \frac{\partial \log(\theta)}{\partial A} + \frac{\partial \log(\theta)}{\partial h}\frac{dh}{dA} + \frac{\partial \log(\theta)}{\partial \hat{\mu}}\frac{d\hat{\mu}}{dA}. \tag{9}$$

Note the following simplifications in Equation 9: for individuals $dz/dA = 0$ by construction as z is innate and non-variable,[9] the α and γ were dropped as they are found to be

[9] While $dz/dA = 0$ for individuals, this is not technically true when thinking about the average worker

small and numerically negligible. The above expression makes clear that $d\log(\theta)/dA$ depends critically on three things: a direct effect from A, a skill effect through h, and a belief effect through $\hat{\mu}$.

Last, before returning to interpret the results of the exercises, it will also be useful to note that the equilibrium sub-market tightness function (and the derivative of its log) is a function of, among other things, h, $\hat{\mu}$, and A. While the expression given in Equation 9 will be appropriated to describe the compositional effects of beliefs and skill-change, much is lost insofar as the individual effects are concerned. To visualize the individual effect, I therefore take and plot the numerical derivative (with respect to A) of the logged equilibrium sub-market tightness function from the calibrated model at the average along the $\hat{\mu}$, h, and A dimensions (at the mean of each of the other variables).

Focusing on the belief-related terms of Equation 9, the calibrated model finds that $\partial log(\theta)/\partial\hat{\mu}$ is negative and, recalling the earlier steady-state exercise, $d\hat{\mu}/dA$ is positive. These opposite signs indicate that uncertainty in job finding has an attenuating compositional effect on the search response to shocks. Beliefs about finding a job are more optimistic in expansions and more pessimistic in recessions. That is, the average sub-market searched in is relatively tighter in recessions and slacker in expansions. Because optimistic beliefs make workers "picky" and search in slack, high-valued sub-markets, the volatility and cyclicality of labor market variables is lower.

What about at the individual level? As we can see in Figure 4, as $\hat{\mu}$ increases, the search response to aggregate shocks gets larger. Intuitively, optimism makes the perceived cost of unemployment lower, compelling workers to respond more to aggregate conditions (and thus less to idiosyncratic ones). To put this into terms which parallels the concept of a reservation wage, workers are more inclined to "wait it out" for better economic conditions. On the one hand, optimistic workers will respond relatively more to positive shocks by searching in tighter sub-markets. On the other, they will also search in relatively slacker sub-markets in response to negative shocks as the assessed cost of doing so is small. That is, volatility and cyclicality measures increase with the degree of optimism.

because of the model's non-linearities. Indeed, because positive shocks generally benefit more productive workers due to the affine production function, $d\bar{z}/dA > 0$. However, this effect is found to be small and, for simplicity of presentation, is dropped.

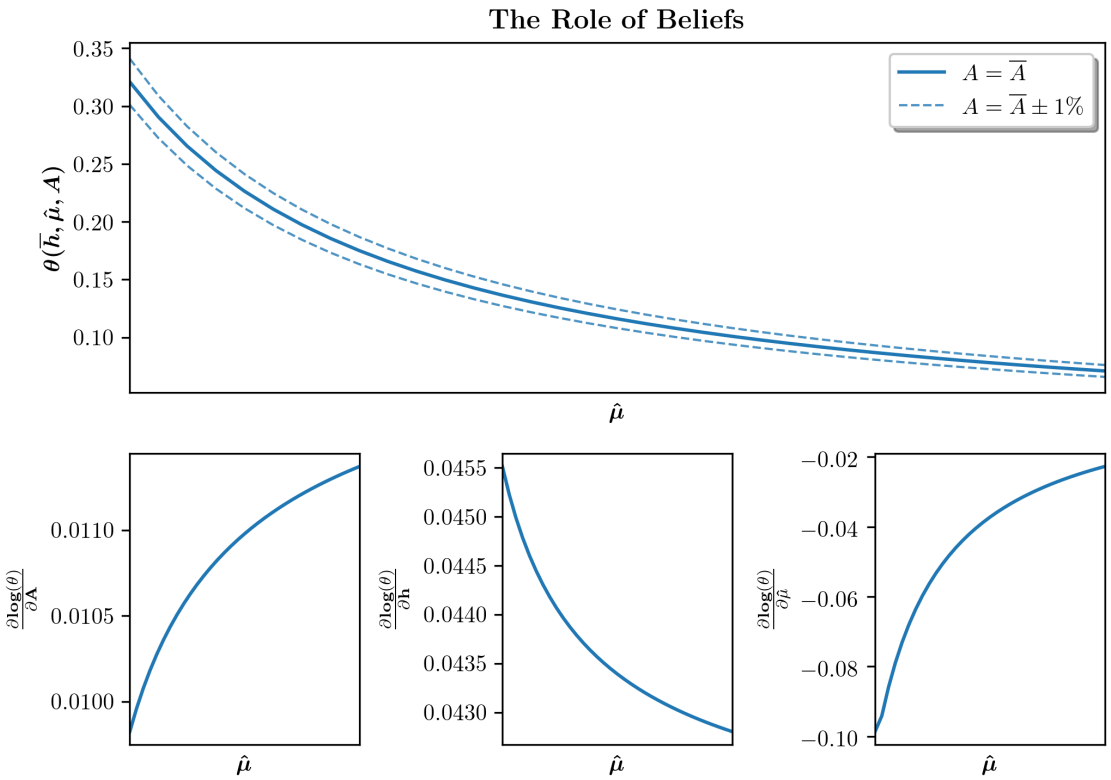

Figure 4: The equilibrium market tightness function and its numerical derivatives plotted across the support of beliefs, evaluated at the mean value of aggregate productivity for the average searcher in the economy. In addition, the market tightness function is also plotted for productivity levels 1% higher and lower than the average.

The skill-related terms of Equation 9, $\partial log(\theta)/\partial h$ and dh/dA are found to be positive, advising that the presence of skill change compositionally augments the search response to aggregate shocks. This is intuitive. Skill loss and accrual result in higher human capital in expansions and lower human capital in recessions. Since firms value higher skilled workers, volatility and cyclicality measures of labor market variables increase because firms post more vacancies in expansions and fewer in recessions. It is also important to note that the model's finding that $\partial log(\theta)/\partial h > 0$ is perhaps not as straightforward as it might seem. As a first pass, it is true that firms are more willing to post vacancies for workers who are more productive. Remember, however, that search is directed. This is to say that workers with lower levels of skills generally choose to search in tighter, low-valued sub-markets. To put this in terms of the demand vs. supply dichotomy, in general equilibrium I find that the demand (for labor) effect dominates: those with higher human capital ultimately search in

tighter sub-markets.

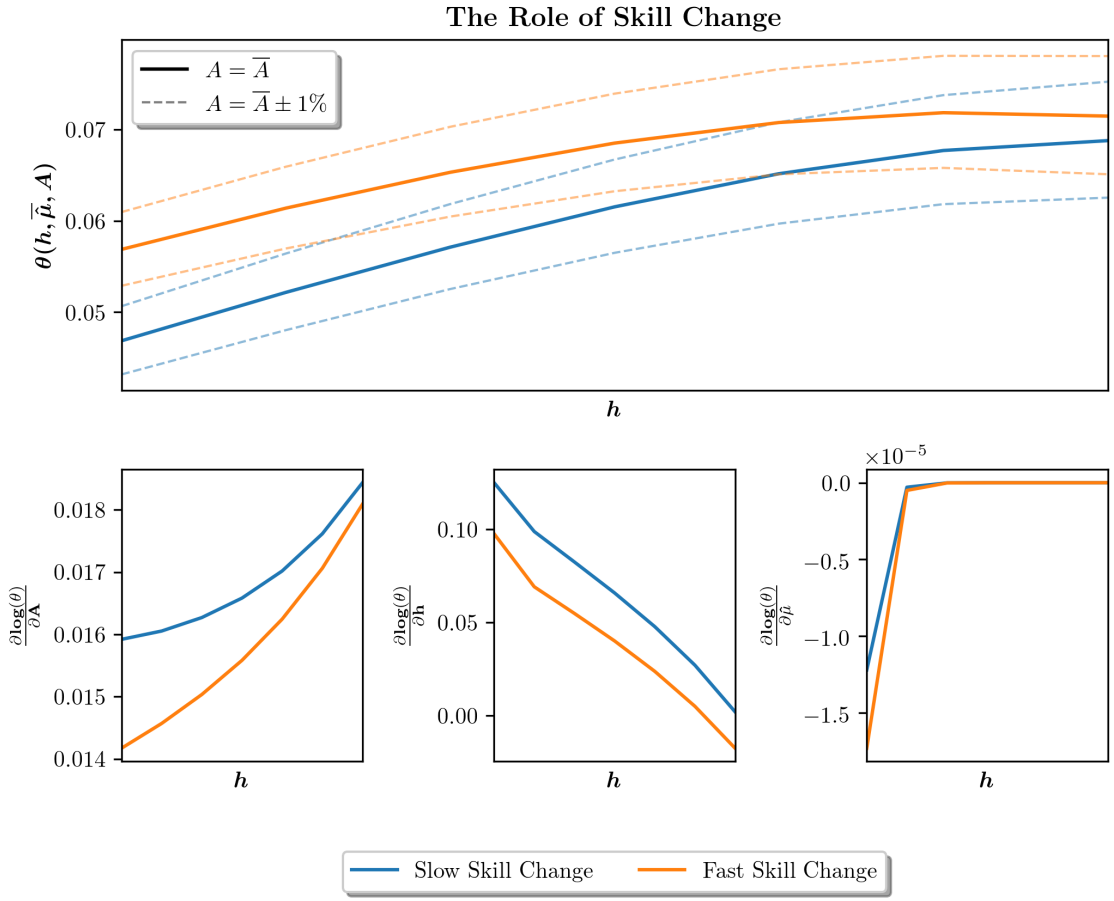

Figure 5: The equilibrium market tightness function and its numerical derivatives plotted across the support of skills, evaluated at the mean value of aggregate productivity for the average searcher in the economy, when skill change is slow (blue) and rapid (orange). In addition, the market tightness function is also plotted for productivity levels 1% higher and lower than the average.

In fact, Figure 5 reveals more about these individual demand and supply effects. Comparing the low (blue) and high (orange) rate of skill change economies, we see that as the rate of skill change is increased, the search response to aggregate shocks dampens the amplification of skill change. From a searching worker's perspective, skill change makes unemployment more costly. Increasing the rate of this change makes these costs even greater as skill decay reduces the desirability of an individual's labor. In response, workers search in tighter sub-markets in order to limit their exposure to skill loss, abating volatility and cyclicality measures. Notice, though, that the degree of this abatement–the vertical distance between

the two curves–is much smaller for high skilled workers. Intuitively, given some h, fast skill accrual makes labor more valuable to firms as they will quickly ascend the ladder. On the upside, firms are more willing to post vacancies when A is high as the potential returns to a filled vacancy are higher because workers can gain skills while working. On the downside, since firms know that workers can accrue skills later on, they are less inclined to post vacancies in a given h sub-market when A is low. Altogether, this demand effect is stronger at higher h because ascending to the top is much quicker, and so the dampening effect of the rate of skill change is–in an abuse of language–dampened the higher up the skill ladder one looks.

Together, the compositional and individual effects for *both* beliefs and skill change move in opposite directions. The calibrated model finds that the individual effects dominate.

7 Conclusion

This paper studies how variable labor market attachment, namely beliefs about job finding and state-dependent skill change, affect the dynamic behavior of labor market variables. These two aspects are shown to be important factors for the labor market decisions of workers. In particular, using the SCE, I find that workers believe that the monthly job-finding probability is twice as high as they realize on-average. To study the dynamic implications, I construct a model of directed search and calibrate it to the US economy.

I find that beliefs about job finding increase the volatility of labor market variables while skill change decreases it. Key to these results is the distinction of compositional and individual effects. That is, how these two aspects of variable worker attachment affect the type of workers searching versus how they affect an individual's choices. Whereas the compositional effects would suggest that beliefs dampen volatility and skill change amplifies it, the individual effects governing where individuals search for work are found to be stronger. This contrasts with models of random search, for example standard DMP models, where workers cannot adjust and directly affect their job-finding probability. There, all effects of beliefs and skill-change occur as a result of changing the bargaining position of workers or the vacancy posting behavior of firms. I find that directed search allows for an additional

margin of adjustment that is important for the aggregate behavior of labor market variables.

The findings of the model suggest an important role for variable worker attachment for the behavior of labor markets. Further, these findings motivate further work on the implications of, specifically, beliefs about job finding. While the present paper abstracts away from the participation margin, the topic of attachment is naturally related to the decision to look for work at all, and is an important extension to explore. Next, a belief mechanism in the presence of on-the-job search may also help to contextualize certain observations in aggregate data, such as the patterns of job-to-job transitions. These additional channels are ostensively important, and left for future research.

8 Appendix

8.1 Proof of Lemma 1

This result is very straightforward and stems from modeling the uncertainty in job finding prospects as beliefs over the scale parameter of the matching technology. Simply writing out the subjective job finding probability for a worker and simplifying will establish this. [10]

$$\hat{f}(\theta) = \int \mu\theta^{\eta} p(\mu) d\mu = \theta^{\eta} \int \mu p(\mu) d\mu = \hat{\mu}\theta^{\eta}, \tag{10}$$

where $\hat{\mu}$ is the prior mean of μ. Since the problem in the future period structurally the same, evaluations of future values of search are similar, i.e. $\hat{f}'(\theta) = \hat{\mu}'\theta^{\eta}$. (Back to Model 4.1)

8.2 Proof of Lemma 2

The proof of this lemma relates the conjugate property of the beta distribution for the likelihood function of a Bernoulli random variable, namely the outcome of job search. Additionally, in the application to a setting with endogenous job finding probabilities that may change over time, a prior mean belief of μ (in addition to α and γ) is necessary to initiate the learning mechanism. First observe that *(i)* is established by noting that the mean of

[10] This, for instance, would not be the case if there were beliefs over the elasticity of the matching function.

$f \sim \text{Beta}(\alpha, \gamma)$, $\alpha \in \mathbb{R}_+$ and $\gamma \in \mathbb{R}_+$, is $\hat{f} = \alpha/(\alpha+\gamma) \in (0,1)$. This mean is increasing (decreasing) in α (γ), and so given $\hat{f}$ there exists a (non-unique) pair (α, γ) that produces it. Bayes' formula establishes *(ii)*. Letting $B(\cdot)$ denote the beta function and $\widetilde{p}(\cdot)$ denote the appropriate probability functions, we have

$$\begin{aligned}
\widetilde{p}(f|y) &= \frac{\widetilde{p}(y|f)\widetilde{p}(f)}{\widetilde{p}(y)} \\
&= \frac{f^y(1-f)^{1-y}\Big[B^{-1}(\alpha,\gamma)f^{\alpha-1}(1-f)^{\gamma-1}\Big]}{B^{-1}(\alpha,\gamma)\displaystyle\int f^{y+\alpha-1}(1-f)^{\gamma-y}df} \\
&= B^{-1}(\alpha',\gamma')f^{\alpha'-1}(1-f)^{\gamma'-1},
\end{aligned} \tag{11}$$

where $\alpha' = \alpha + y$ and $\gamma' = \gamma + 1 - y$. Finally, *(iii)* may be backed out from the expression for the posterior mean of f.

$$\begin{aligned}
\hat{f}' &= \frac{\alpha'}{\alpha'+\gamma'} \\
\hat{\mu}'\theta^{\eta} &= \frac{\alpha'}{\alpha'+\gamma'} \\
\hat{\mu}' &= \left(\frac{\alpha'}{\alpha'+\gamma'}\right)\theta^{-\eta}
\end{aligned} \tag{12}$$

(Back to Model 4.1)

8.3 Proof of Proposition 1

Consider an unemployed worker who is searching for work with résumé ϕ and aggregate state ψ as given. Write the sub-market tightness function of concern for this worker as $\theta_x = \theta(x|\phi,\psi)$. Though workers do not know the true likelihood with which they find jobs, it is assumed that they do observe the vacancy posting behavior of firms. The value of a job that a worker searches for can be written as a function of the sub-market tightness by rearranging Equation 5:

$$x = \mathbb{E}_{\psi'|\psi}\sum_r \mathcal{V}_E(z, h'_r, p_1, \psi')\pi_u(h'_r|h) - \frac{\kappa}{q(\theta_x)}. \tag{13}$$

The above expression makes clear the trade-off faced by searchers: higher value jobs come at the cost of slack sub-markets. The above can also be used to rewrite the search problem of an unemployed worker in terms of a choice of market tightness,

$$\max_{\theta_x} \left\{ \hat{f}(\theta_x) \mathbb{E}_{\psi'|\psi} \sum_r \Big[\mathcal{V}_E(z, h'_r, p_1, \psi') - \mathcal{V}_U(z, h'_r, p_0, \psi') \Big] \pi_u(h'_r|h) - \frac{\hat{\mu}}{\mu} \theta_x \kappa \right\}. \tag{14}$$

Under the assumption of a Cobb-Douglas matching technology, the solution to the above problem is

$$\theta_x = \left\{ \frac{\eta\mu}{\kappa} \mathbb{E}_{\psi'|\psi} \sum_r \Big[\mathcal{V}_E(z, h'_r, p_1, \psi') - \mathcal{V}_U(z, h'_r, p_0, \psi') \Big] \pi_u(h'_r|h) \right\}^{\frac{1}{1-\eta}}. \tag{15}$$

Substituting Equation 15 into Equation 13 yields an expression for the searching worker's policy.

$$x(z, h, p, \psi) = \mathbb{E}_{\psi'|\psi} \sum_r \Big[(1-\eta) \mathcal{V}_E(z, h'_r, p_1, \psi') + \eta \mathcal{V}_U(z, h'_r, p_0, \psi') \Big] \pi_u(h'_r|h) \tag{16}$$

Since the RHS of Equation 16 is increasing in $\hat{\mu}$, there exists a one-to-one relationship between x and $\hat{\mu}$ given ϕ and ψ. Thus, firms offering x to workers with ϕ and ψ observable can back out $\hat{\mu}$ whenever they meet someone in the labor market. (Back to Model 4.3)

8.4 Proof of Theorem 1

Let $\mathcal{V} : \{0, 1\} \times \mathbb{R}_+ \times H \times P \times \Psi \to \mathbb{R}$ be a function defined such that $\mathcal{V}(0, z, h, p, \psi) = \mathcal{V}_U(z, h, p, \psi)$ and $\mathcal{V}(1, z, h, p, \psi) = \mathcal{V}_E(z, h, p, \psi)$. Equation 3, and Equation 4 can then be rewritten as

$$
\begin{aligned}
\mathcal{V}(a,z,h,p,\psi) =& a\Bigg[Azh + \beta\mathbb{E}_{\psi'|\psi}\max_s\Big\{s\sum_r \mathcal{V}(0,z,h'_r,p,\psi')\pi_e(h'_r|h) \\
&+ (1-s)\sum_r \mathcal{V}(1,z,h'_r,p,\psi')\pi_e(h'_r|h)\Big\}\Bigg] \\
&+ (1-a)\Bigg[b + \beta\mathbb{E}_{\psi'|\psi}\max_x\Big\{\hat{f}(\theta_{x,\phi,\psi})x \\
&+ \big(1-\hat{f}(\theta_{x,\phi,\psi})\big)\sum_r \mathcal{V}(0,z,h'_r,p_0,\psi')\pi_u(h'_r|h)\Big\}\Bigg].
\end{aligned} \tag{17}
$$

Next, note that the worker's choice of x can be written in terms of θ, z, h, p, and ψ using Equation 5 as $x(\theta,z,h,p,\psi) = \mathbb{E}_{\psi'|\psi}\sum_r \mathcal{V}(1,z,h'_r,p_1,\psi')\pi_u(h'_r|h) - \frac{\kappa}{q(\theta)}$. As in Menzio & Shi (2011), x cannot be uniquely expressed as a function of market tightness and the state variables in sub-markets with $\theta_{x,\phi,\psi} = 0$. However, this is irrelevant insofar as the worker can never find a job in these sub-markets. As such, it can be assumed w.l.o.g. that those sub-markets with $\theta = 0$ have values given by the above. Now substitute $x(\theta,z,h,p,\psi)$ for x and θ for $\theta_{x,\phi,\psi}$ in Equation 17 and write the problem as follows.

$$
\begin{aligned}
\mathcal{V}(a,z,h,p,\psi) =& a\Bigg[Azh + \beta\mathbb{E}_{\psi'|\psi}\max_s\Big\{s\sum_r \mathcal{V}(0,z,h'_r,p,\psi')\pi_e(h'_r|h) \\
&+ (1-s)\sum_r \mathcal{V}(1,z,h'_r,p,\psi')\pi_e(h'_r|h)\Big\}\Bigg] \\
&+ (1-a)\Bigg[b + \beta\mathbb{E}_{\psi'|\psi}\max_\theta\Big\{\hat{f}(\theta)\sum_r \mathcal{V}(1,z,h'_r,p_1,\psi')\pi_u(h'_r|h) \\
&- \frac{\hat{\mu}}{\mu}\kappa\theta + \big(1-\hat{f}(\theta)\big)\sum_r \mathcal{V}(0,z,h'_r,p_0,\psi')\pi_u(h'_r|h)\Big\}\Bigg].
\end{aligned} \tag{18}
$$

Let $\Omega = \{0,1\}\times\mathbb{R}_+\times H\times P\times\Psi$ and let $C(\Omega)$ denote the space of bounded continuous functions $R:\Omega\to\mathbb{R}$, with the sup norm. Let $T: C(\Omega)\to C(\Omega)$ be the operator associated with Equation 18. The following can be easily established.

(i) T is monotonic: for $R_1, R_2 \in C(\Omega)$ where $R_1 \le R_2$ w.l.o.g., $T(R_1) \le T(R_2)$.

(ii) T discounts: for $R \in C(\Omega)$ and $c \in \mathbb{R}_+$, $T(R + c) = TR + \beta c$.

Thus, by Blackwell's sufficiency conditions, we have that the operator T is a contraction mapping and there exists a unique fixed point $\mathcal{V}$. Next, it is also easy to see that R depends on ψ' only through A'. It is thus also the case that $T(R)$ depends on ψ only through A. This logic similarly applies to the agents' policy functions. Thus the value and policy functions only depend on the aggregate state through realizations of the aggregate shock, A, and not on the distribution of workers (and their résumés) across employment states. (Back to Model 4.3)

8.5 Bilaterally Efficient Contract with Constant Wages

Suppose a worker finds a job that pays a fixed, non-changing wage w each period with separation contingency specified in the employment contract given by s. Her lifetime utility can be written as

$$\begin{aligned}\mathcal{V}_W(w; z, h, p, \psi) = w + \beta \mathbb{E}_{\psi'|\psi} \sum_r \Big[& s(z, h'_r, p, \psi')\mathcal{V}_U(z, h'_r, p, \psi') \\ & + (1 - s(z, h'_r, p, \psi'))\mathcal{V}_W(w; z, h'_r, p, \psi') \Big] \pi_e(h'_r|h).\end{aligned} \tag{19}$$

The fixed-wage is given by the solution to the above where $x = \mathcal{V}_W(w; z, h, p, \psi)$, where x is the value promised by the firm to the worker when the contract was signed. It should also be noted that all block recursive results established above also apply here. That is, $\mathcal{V}_W$ can be shown to depend on ψ only through A. For a formal treatment of fixed-wage (and other) contracts in directed search, block recursive environments, see Menzio and Shi (2010a). (Back to Calibration 5)

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