Informal Caregiving, Family Power Dynamics, and Labor Market Rigidities

Preliminary Draft, Please Do Not Circulate.

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Abstract

Do informal caregivers bear the entire cost of caregiving? Standard cooperative models of the household suggest the welfare burden of care would be distributed across household members (e.g. husband and wife). We develop and calibrate a simple collective model of intra-household bargaining to analyze the time and resource allocation decisions associated with informal care. We show that a decrease in bargaining power increases one's share of the welfare burden. If bargaining power is endogenously determined by relative earnings, the welfare cost of caregiving can fall disproportionately on a single partner, resulting in a "triple burden" of market work, home production, and caregiving. Labor market rigidities exacerbate the total welfare cost of informal caregiving to the household as well as the unequal distribution of the burden.

1 Introduction

A growing concern in many countries is an aging population and an increase in the number of elderly in need of long-term care (LTC). However, the economic and welfare implications of LTC provision remain relatively understudied. One of the primary factors which complicates welfare analysis is that a bulk of elderly care is provided by family members. Norton (2000) estimates that roughly two-thirds of LTC is provided informally. The pervasiveness of informal care due to cultural or family ties has even limited the development of LTC insurance in economically advanced regions like Europe (Costa-Font, 2010). Hence, in order to understand the full macroeconomic implications of growing LTC demands and the appropriate policy response, it is imperative to explore how households cope with these caregiving needs.

Informal caregiving is a time intensive task and must be met by adjustments along leisure or work margins on the part of the care provider. But do caregivers share the full burden of care with other members of their household? Consider a couple (e.g. husband and wife) living together in a household and providing LTC for an elderly parent. If the couple shares caregiving tasks evenly, then the provision of LTC could simply imply a reduction in overall time available to the couple without necessarily changing the distribution of welfare within the household. However, existing literature shows that caregiving falls disproportionately on women as compared to men (Barusch and Spaid, 1989). There is also some evidence that caregivers respond to these shocks by reducing work hours, taking more flexible jobs, or by exiting the labor market completely (Bauer and Sousa-Poza, 2015). This implies a redistribution of time across work (potentially market and non-market), leisure, and caregiving within the household. However, how this redistribution effects relative welfare across the couple is unclear.

In this study, we develop a theoretical model to explore the implications of informal caregiving on the distribution of welfare within a couple under different assumptions regarding household power dynamics and labor market flexibility. Existing models of LTC have generally focused on inter-generational bargaining between parents and children or bargaining among siblings (e.g. Pezzin and Schone, 1999; Engers and Stern, 2002; Byrne et al., 2009; Barczyk and Kredler, 2017). However, little attention has been paid to the influence of LTC demands on the power dynamics between partners within a household. In the standard "unitary model" pioneered by Becker (1981), a household behaves *as if* it were a single unit. In contrast, the pooling of resources across the household implied by the unitary model has been consistently and repeated rejected empirically (e.g. Lundberg et al., 1997; Fortin and Lacroix, 1997). We use a simple collective bargaining model of intra-household time and resource allocation in the spirit of Chiappori (1992). This approach views partners as individuals with conflicting preferences but that operate as a cooperative decision making unit.

We compare theoretical and numerical results under three alternate modeling assumptions. (1) *Exogenous bargaining power* for each partner and fully flexible labor markets. We use this as our baseline to examine how informal caregiving changes the allocation of time and consumption within a household under a given bargaining power structure. (2) *Endogenous bargaining power* dependent on the decisions of the household. Specifically, we assume bargaining position is tied to relative earnings. A decline in labor earnings in response to

informal caregiving can potentially weaken the caregiver's bargaining position and affect the distribution of resources within the couple. Thus LTC provision may have additional implications for gender inequality if we take into account changing power dynamics within the household. (3) *Fixed labor supply* due to labor market rigidities. In the presence of labor market rigidities, any adjustments on the labor supply margin can be costly. Under such circumstances, caregiving needs must be met by adjustments solely along the leisure or home production margins.

We calibrate our theoretical model using cross-country data from the Survey of Health, Ageing, and Retirement in Europe (SHARE). In our quantitative exercise, we find that for a representative couple in our benchmark country (France), 15 hours a week of informal care results in a 15 percent welfare decline for men and 16 percent for women under the assumption of exogenous bargaining power. In other words, the welfare burden of informal caregiving to the male is about 95% that of their female partner—a fairly even distribution. Comparing across countries, in those with the highest level of calibrated female bargaining power (e.g. Sweden), the welfare burden can even fall more heavily on the male than the female, despite the female providing all the informal care. This is a result of higher average labor market productivity of men driving an increase in male labor supply and a decrease in female labor supply.

In contrast to the fairly even distribution under exogenous bargaining, the welfare cost of caregiving can become highly skewed against the caregiver when there is endogenous bargaining power. In our numerical example, the relative welfare cost for the French male falls from 95% to only 4% of that of the female. More generally, countries with lower baseline female bargaining weights see the largest shift of the welfare burden towards the female caregiver. There are two related reasons for the shift in welfare burden. First, declines in female labor supply reduce the bargaining power of the female. Second, the female labor supply response is smaller due to the negative influence on bargaining power.

Lastly, allowing for labor market rigidities also results in significant welfare differentials within a household. For instance, when both men and women are unable to adjust work hours, the relative welfare cost to the male in France is 20% of that of the female. However, for a more equal country like Sweden, the relative welfare cost to the male is 48%. Moreover, only in the most equal countries in our sample do the partners split caregiving responsibilities. In most countries, the female continues to provide the entire amount of informal care, even when adjustments on the formal labor supply margin are eliminated. These are also the only countries where men increase home production when women provide informal care.

Our theoretical and numerical results show that ignoring bargaining power differentials can misrepresent the welfare effect of informal caregiving by not taking into account the uneven distributional consequences. In our model, a decrease in bargaining power increases one's share of the welfare burden. If bargaining power is endogenously determined by relative earnings, the welfare cost of caregiving can fall disproportionately on a single partner, resulting in a "triple burden" of market work, home production, and caregiving. Under this scenario, government policies subsidizing long-term care could decrease the welfare gap within a household by providing financial relief *and* improving the bargaining position of the caregiver. In general, labor market rigidities exacerbate the total welfare cost of informal caregiving to the household as well as the unequal distribution of the burden. This implies policies that promote flexibility, such as caregiver leave, could provide substantial relief,

particularly to high intensity caregivers.

2 Model

Consider a household consisting of two working adults. For expositional convenience, we refer to household partners as female and male. Each member has their own utility function designating preferences over own consumption of market goods (c), domestically produced goods (d), and leisure (l). Utility is separable and given by:

$$u\left(c_i, d_i, l_i\right) = \log c_i + \kappa \log d_i + \phi\left(l_i\right)$$

where $\phi' > 0$ and $\phi'' < 0$. Each member is endowed with a unit of time that is split between work in the formal labor market (e), hours devoted to domestic home production (h), and leisure (l):

$$e_i + h_i + l_i = 1. \tag{1}$$

The household budget constraint is given by:

$$c_m + c_f \quad = \quad e_m + \gamma e_f \tag{2}$$

where γ denotes the potential earnings differential between household members. Partners combine productive home hours to produce domestic goods using a constant elasticity of substitution technology:

$$d_m + d_f \quad = \quad \left(\alpha h_m^{\eta} + (1-\alpha)\, h_f^{\eta}\right)^{\frac{1}{\eta}} \tag{3}$$

with $\alpha \in (0,1)$ and $\eta < 1$.

Following the collective bargaining approach of Chiappori (1992), the Pareto-efficient allocations are derived by maximizing the waited sum of partner utilities. Specifically, the partners maximize the household welfare function:

$$(1-\theta)\, u\left(c_m, d_m, l_m\right) + \theta u\left(c_f, d_f, l_f\right)$$

subject to constraints (1)-(3). Here, $\theta \in [0,1]$ measures the relative bargaining power of the female in the household. Note that with equal bargaining weights (a parameter value $\theta = 0.5$), the collective bargaining allocation reduces to that of the standard unitary model.

2.1 Exogenous bargaining power

First consider the case where bargaining power θ is exogenous to the household. Bargaining power is likely influenced by cultural norms, prevailing female earnings potential, local institutions, and a variety of other external factors. In this case, the optimal household consumption allocations follow a "sharing rule" as is typical in this type of collective bargaining model and stated in the following proposition.

Proposition 1. *Each partner consumes a fraction of household income and domestic production equal to their bargaining weight:*

$$c_m = (1-\theta)(e_m + \gamma e_f), \quad c_f = \theta(e_m + \gamma e_f) \tag{4}$$

$$d_m = (1-\theta)\left(\alpha h_m^\eta + (1-\alpha) h_f^\eta\right)^{\frac{1}{\eta}}, \quad d_f = \theta\left(\alpha h_m^\eta + (1-\alpha) h_f^\eta\right)^{\frac{1}{\eta}} \tag{5}$$

Proof. See appendix. □

Turning next to female labor supply, the relevant first-order condition is given by:

$$\frac{\gamma}{e_m + \gamma e_f} = \theta \phi'(1 - e_f - h_f). \tag{6}$$

The household equates the marginal benefit of female labor to the weighted marginal cost. As an illustrative example, Figure 1 provides a graphical representation of equation (6), holding home production hours and male labor supply constant. We will refer to this curve as the labor supply curve. The curve is downward sloping, reflecting the fact that lower bargaining power increases female labor supply.[1] As is the clear from the figure, for a given set of parameters, a bargaining weight $\theta < 0.5$ results in higher female labor supply than the corresponding unitary model. Intuitively, as female bargaining power rises, she is afforded more leisure by lowering formal labor supply and time devoted to home production. The inverse is true for the male partner—increases in female bargaining weight θ increases male labor supply. These results are more generally stated in the following proposition.

Proposition 2. *Comparative statics for labor supply response to changes in exogenous bargaining power are given by:*

$$\frac{\partial e_f}{\partial \theta} < 0, \quad \frac{\partial e_m}{\partial \theta} > 0.$$

Proof. See appendix. □

While bargaining power has clear implications for relative labor supplied across partners, this is not the case for hours devoted to domestic production. Combining household optimality conditions yields the follow rule for relative allocation of home production hours across partners:

$$h_m = \left[\frac{\gamma\alpha}{1-\alpha}\right]^{\frac{1}{1-\eta}} h_f. \tag{7}$$

This implies male home production hours will be lower than female whenever $\gamma < \frac{1-\alpha}{\alpha}$. Intuitively, when female market returns γ are small relative to their home production share α, females will spend more time at home relative to the male partner. Note also that relative

[1] One could also envision an alternate case where female labor supply *increases* with bargaining power if the male has a strong preference for the female to limit market work. This may be the case in more traditional or conservative societies. We focus on the standard case with downward sloping female labor supply curve. In our numerical exercise, we focus on European couples where both partners have strong labor force attachment, where the standard curve is likely to hold.

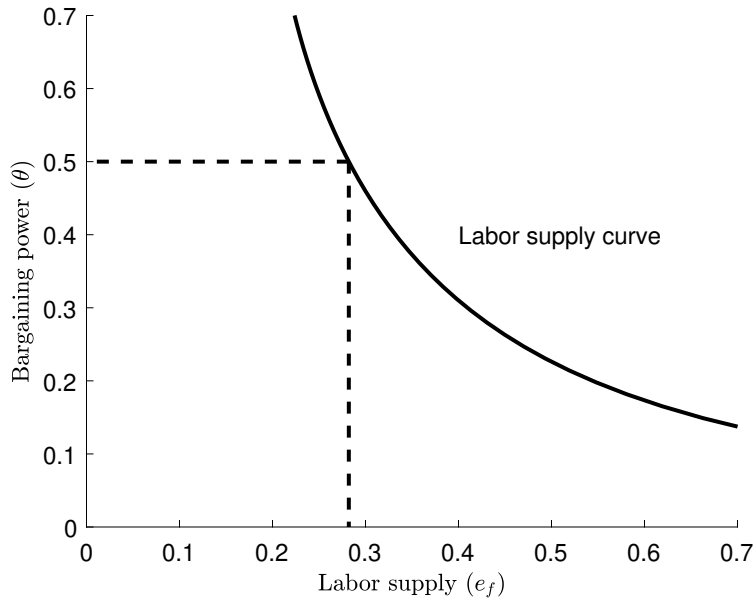


Figure 1: Bargaining power and female labor supply

home production does not depend on bargaining power weight θ, but only on the relative productivity of home versus market work.

Finally, we can summarize the home and market work burden distribution across partners through the following condition:

$$\phi'(l_m) = \frac{\theta}{(1-\theta)\gamma}\phi'(l_f). \tag{8}$$

This shows that the male partner will enjoy more leisure than the female whenever $\frac{\theta}{1-\theta} < \gamma$. Conversely, if the female market return γ is low relative to her bargaining weight θ, she will enjoy more leisure.

2.2 Informal caregiving

We next consider the household equilibrium response to the provision of informal care. Clearly there are plausible arguments for treating the provision of informal care as a complex endogenous decision involving cultural norms, bargaining between caregiver and care recipient, and/or bargaining between potential caregivers (e.g. siblings). However, as our focus is on time allocation decisions among actual caregivers, we abstract from such consideration and simply treat informal care as exogenous to the household time endowment $z \in (0, 1)$. This assumption allows us to examine the strength of the intra-household bargaining mechanism of interest. Moreover, several studies have found only weak evidence of endogenous caregiving decisions, particularly after controlling for individual fixed effects (Bolin et al., 2008; Ciani, 2012; Meng, 2013; Van Houtven et al., 2013; Nguyen and Connelly, 2014).

With informal care, the modified time constraint for each partner and feasibility require-

ments on caregiving are given by:

$$e_i + h_i + l_i + z_i = 1$$
$$z_f + z_m = z, \quad z_f, z_m \geq 0.$$

Note that we assume informal care time inputs are perfect substitutes across partners. In this case, it is optimal for the partner with lower market return to provide all informal care.

Proposition 3. *If $e_f > 0$ and $\gamma < 1$, the female partner will provide all the informal care in the household: $z_f = z$ and $z_m = 0$. If $e_m > 0$ and $\gamma > 1$, the male partner will provide all the informal care in the household: $z_f = 0$ and $z_m = z$.*

Proof. See appendix. □

So long as the female partner is less productive in market work, it is optimal for her to specialize in informal care. Moreover this specialization comes at the cost forgone market work and home production for the female.

Proposition 4. *If $\gamma < 1$, interior comparative statics for labor supply and home production response to changes in informal caregiving are given by:*

$$\frac{\partial e_f}{\partial z} < 0, \quad \frac{\partial h_f}{\partial z} < 0, \quad \frac{\partial e_m}{\partial z} > 0, \quad \frac{\partial h_m}{\partial z} < 0.$$

Proof. See appendix. □

In contrast to the female, the male partner increases market work in response to increasing informal care. However, the male lowers his home production in the same fashion as the female. This is due to the assumption that home inputs are less then perfect substitutes across partners. Therefore, lower home production from the female decreases the marginal return to male home production. Thus the male substitutes some hours out of home production and into market labor. Moreover, condition (7) continues to hold in the presence of increasing informal care. So not only do male and female home production move in the same direction, they remain in the same proportions.

2.3 Endogenous bargaining power

It is plausible that bargaining power may not only depend on prevailing external factors, but could also endogenously evolve depending on the decisions of the household. In particular, theoretical and empirical research suggests that one's relative earnings within the household may play an important role (Mencher, 1988; Blumberg and Coleman, 1989; Desai and Jain, 1994; Riley, 1997; Lundberg et al., 1997; Attanasio and Lechene, 2002; Bonke and Browning, 2009). In this case, we may think θ is dependent on the distribution of income within the household (Basu, 2006).

In this spirit, we consider an endogenously determined bargaining power defined by $\theta = \theta(E)$, where $E = \frac{\gamma e_f}{e_m}$ is the female to male earnings ratios. We assume $\theta'(E) \geq 0$ so that bargaining power is increasing in relative earnings. For a *given* value of θ, the household maximizes their previously defined welfare function. However, this may in turn cause θ to

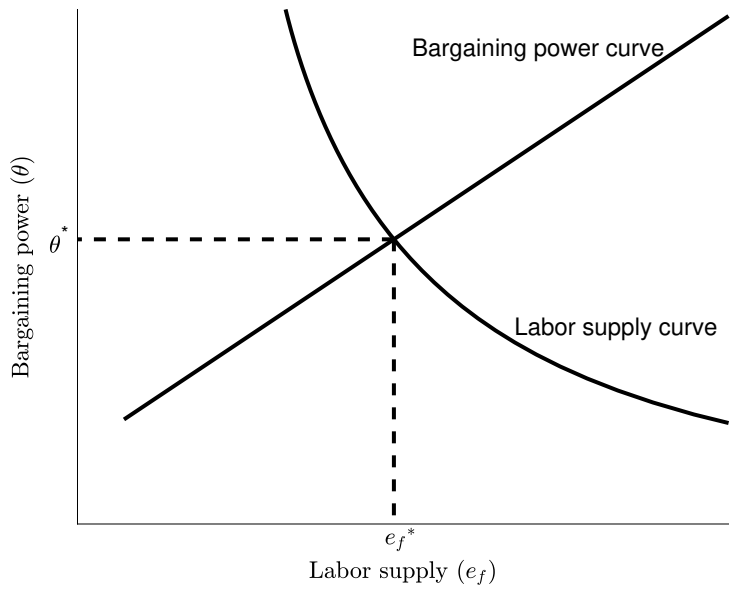


Figure 2: Endogenous bargaining power and female labor supply

change, resulting in further desired adjustments. Following the idea of Basu (1999; 2006), we consider the stationary point of this process as the equilibrium of interest. Denoting the solution to the household maximization problem given θ as $q(\theta) = \{c_m, c_f, d_m, d_f, e_m, e_f, h_m, h_f\}$, the equilibrium of this adjustment process can be defined as follows:

Definition 5. A household equilibrium with endogenous bargaining power is a vector of outcomes q^* and a power index θ^*, such that $\theta^* = \theta(E^*)$, and $q^* = q(\theta^*)$, where E^* is the earnings ratio arising from outcome vector q^*.

Following this definition and using the household's first-order condition (6), a female labor supply allocation e_f^* is part of an equilibrium if

$$\frac{\gamma}{e_m^* + \gamma e_f^*} = \theta(E^*)\,\phi'\left(1 - e_f^* - h_f^*\right). \tag{9}$$

As an illustration, Figure 2 plots the labor supply curve defined by (6) along with a hypothetical endogenous bargaining power curve $\theta(E)$—holding male labor supply constant. The bargaining power curve is upward sloping in female labor supply as more market work increases the female's relative earnings. The equilibrium described by (9) is given by the intersection of the two curves.

How does incorporating endogenous bargaining power change the predictions of the model when informal care is introduced? Incorporating the care requirement z into the household welfare function yields a modified first-order condition for female labor supply:

$$\frac{\gamma}{e_m + \gamma e_f} = \theta\phi'(1 - e_f - h_f - z_f). \tag{10}$$

Figure 3 plots the female labor supply curve with and without informal caregiving. Introducing informal care shifts the labor supply curve to the left. Consider starting in the

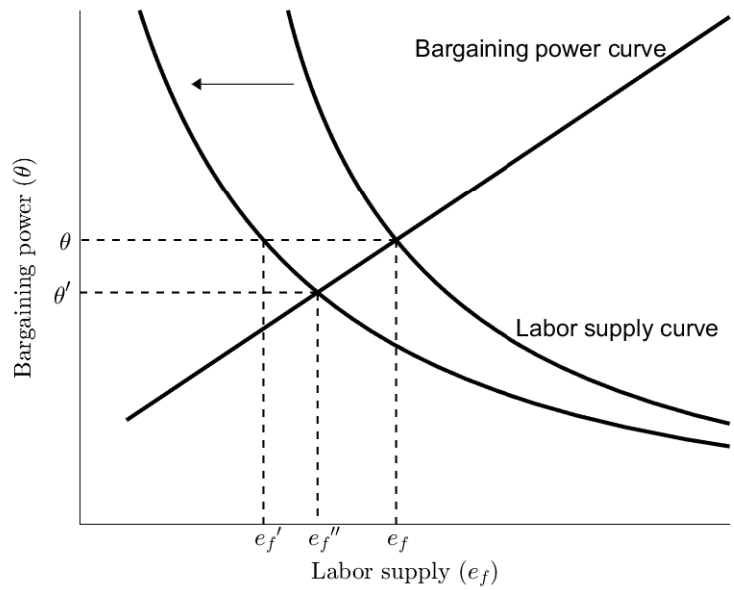


Figure 3: Female labor supply reponse to informal care shock

equilibrium of the non-caregiving household (e_f, θ). In the case of exogenous bargaining power, the caregiving requirement results in new equilibrium $\left(e_f', \theta\right)$. In contrast, the new endogenous bargaining power equilibrium occurs at point $\left(e_f'', \theta''\right)$, where the new labor supply curve intersects the power curve. Given the upward slope of the power curve, it is clear that $\theta' > \theta''$ and $e_f' < e_f''$. The female labor supply response to informal caregiving is weaker with endogenous bargaining power. The magnitude of this difference will depend on the shape and slope of the power curve.

A weak labor supply response is generally consistent with empirical evidence that suggests there is a negative but often relatively small effect of caregiving on hours worked (Bauer and Sousa-Poza, 2015). Several mechanisms have been proposed as potential explanations for the small labor response. These include a countervailing wealth effect due to either increased expenses associated with providing informal care (e.g. food, medicine, etc.) or wage declines due to less work flexibility (Twigg and Atkin, 1994; Heitmueller and Inglis, 2007). A "respite effect" has also been proposed where informal caregivers prefer work in order to get away from their caregiving responsibilities (Twigg and Atkin, 1994). Here we are considering an additional complementary mechanism that operates through the distribution of bargaining power in the household. The next section also explore the possibility that labor markets are rigid, and thus some caregivers (and their partners) are simply unable to adjust along the intensive margin.

This simple framework makes sharp predictions about the difference in labor supply response to informal caregiving given exogenous versus endogenous bargaining power. In contrast, the difference in labor supply response between the unitary and exogenous bargaining power models is theoretically ambiguous. More generally, whether the magnitude of the labor supply response increases or decreases with an exogenously given θ will depend on functional forms and parameter values. In our calibrated numerical example we show that

differences can feasibly be quite large.

2.4 Fixed labor supply

Empirical evidence suggests informal caregiving lowers labor supply on both the extensive and intensive margins. However, the effects of caregiving on employment and hours worked are often found to be relatively small overall, particularly for low intensity care (Bauer and Sousa-Poza, 2015). Our model has so far assumed fully divisible labor and frictionless labor markets. This allows partners to freely allocate hours between market work, home production, and leisure. If females could only operate on the extensive margin, they may be unable to optimally lower their labor supply in the presence of informal caregiving requirements. Likewise, the desired increase in male labor hours may be infeasible if additional overtime or shifts are unavailable. Moreover, if there are re-employment costs and informal caregiving is unlikely to persist for an extended period of time, females may be unwilling to lower labor supply to the same extent as the static frictionless model predicts. This may be why becoming a caregiver reduces labor force participation but leaving the caregiver role has no effect on the probability of re-entry into the labor market (Spiess and Schneider, 2003; Wakabayashi and Donato, 2005; Van Houtven et al., 2013). In this section, we examine the theoretical implications of assuming fixed labor supply in our model.

An important implication of fixed labor supply is that informal caregiving need no longer optimally fall entirely to the female partner. Given the optimal allocation of home production with fixed labor supply, the relevant first-order condition for informal care allocation is given by

$$\theta\phi'\left(1-e_f-h_f-z_f\right)=\left(1-\theta\right)\phi'\left(1-e_m-h_m-z_m\right). \tag{11}$$

Partners divide caregiving time to equalize their marginal cost of lost leisure, weighted by relative bargaining power. With large enough care requirement z, it is possible that this interior solution is reached, and partners share caregiving responsibilities. This contrasts the endogenous labor supply model, where the female always provides full care (under the conditions of Proposition 3).

At an interior solution, home production allocation with fixed labor supply is determined by the following modified condition

$$h_m=\left[\frac{\alpha}{(1-\alpha)}\right]^{\frac{1}{1-\eta}}h_f. \tag{12}$$

Comparing to the analogous condition with endogenous labor (7), the optimal distribution of home production is still independent of bargaining weights. However, this allocation no longer depends on market productivity γ, as the labor supply margin is eliminated. Instead, the distribution is entirely determined by home production technology parameters.

If the non-negativity constraint on informal care binds, and caregiving falls to a single partner, equation (11) need no longer hold. In this case, the home production allocation condition becomes

$$h_m=\left[\frac{\alpha\theta\phi'\left(l_f\right)}{\left(1-\alpha\right)\left(1-\theta\right)\phi'\left(l_m\right)}\right]^{\frac{1}{1-\eta}}h_f.$$

Relative home production now depends on bargaining power with male hours increasing in θ. Intuitively, as home hours are imperfect substitutes, it is optimal to use bargaining power to adjust time along other more substitutable dimensions (i.e. work or caregiving). However, with fixed labor supply and binding non-negativity constraint on care, the only margin left to utilize one's bargaining weight is home production.

3 Numerical example

We next explore the quantitative implications of the simple bargaining power model by calibrating parameters and simulating the influence of exogenous informal caregiving on consumption and time allocations within a household. We begin with a more detailed analysis of a representative household from a single country (France) before conducting a more general cross-country comparison.

3.1 France

3.1.1 Calibration

In order to numerically calibrate our model, we use data primarily from the Survey of Health, Ageing, and Retirement in Europe (SHARE), a longitudinal study of individuals aged 50 or older and their partners covering 27 European countries and Israel. SHARE data contains information on socioeconomic-status and social and family networks, including labor market outcomes and time spent in informal caregiving. There are currently six waves of SHARE available, collected biennially between 2004 and 2017. We use SHARE data on gender, age, country of residence, weekly hours worked, earnings, and caregiving.

As our framework incorporates only limited heterogeneity, we restrict the SHARE sample to as homogeneous a population as feasible for our exercise. After pooling across all survey waves, we retain observations for individuals aged 40-59 who live with a partner. As most informal caregiving (excluding to one's partner) is provided to parents, this age range captures a large share of caregivers. Limiting the sample to those under 60 also lessens concerns over simultaneous retirement and caregiving decisions. We define an informal caregiver as anyone that reported giving personal care or practical household help "about daily" to someone in the previous twelve months.[2] We calibrate parameters using wage and hours worked SHARE data from couples in which both partners are non-caregivers working at least 20 hours a week and neither partner requires care themselves. In other words, we attempt to calibrate the model to a representative household with no caregiving and in which both partners are substantially attached to the labor market.

For our numerical exercise, we first set relative female earnings potential $\gamma = 0.76$ to equal the aggregate wage ratio of females to males in France from our SHARE sample.[3] We then define standard preferences over leisure given by:

$$\phi(l) = \frac{\nu\epsilon}{1+\epsilon}(1-l)^{\frac{1+\epsilon}{\epsilon}},$$

[2]This includes reportedly helping others outside or inside the household.

[3]Hourly wage calculates as reported annual earnings divided by reported weekly hours worked times 52.

Table 1: Calibration for France

Parameter		Value	Target		Source
Relative earnings	γ	0.76	Gender wage ratio	0.76	SHARE
Frisch elasticity	ϵ	1.00	Frisch elasticity	0.5-2.0	Literature
Utility weight on l	ν	6.10	Male hours worked	42.5	SHARE
Female bargaining power	θ^*	0.42	Female hours worked	37.1	SHARE
Home hours substitutibility	η	0.49	Male home hours	11.9	MTUS
Utility weight on d	κ	0.39	Female home hours	20.5	MTUS
Home hours male share	α	0.50	Assumption	—	—

where ϵ is a constant Frisch elasticity of labor supply (the elasticity of labor supply with respect to wage, holding the marginal utility of consumption fixed). Empirical studies of the Frisch elasticity vary considerably, with estimates ranging from 0.5 to nearly 2 (Hall, 2009; Chetty, 2012). We choose a value of $\epsilon = 1$. Given this form, equations (6) and (8) can be written

$$\nu = \frac{\gamma\left(e_f + h_f\right)}{\theta\left(e_m + \gamma e_f\right)} \tag{13}$$

$$\theta = \frac{1}{\frac{e_f + h_f}{\gamma(e_m + h_m)} + 1}. \tag{14}$$

For home production technology, we assume $\alpha = 0.5$, implying partners are equally productive in the production of home goods. Condition (7) may then be written:

$$\eta \;=\; 1 - \frac{\log \gamma}{\log \frac{h_m}{h_f}}. \tag{15}$$

Finally, combining household first-order conditions for labor supply and home production yields the follow equation for utility weight on home consumption:

$$\kappa = \frac{\left(1 + \gamma^{\frac{\eta}{\eta - 1}}\right) h_m}{e_m + \gamma e_f}. \tag{16}$$

Equations (13)-(16) express the four remaining parameters $\{\nu, \theta, \eta, \kappa\}$ as functions of four equilibrium moments $\{e_f, e_m, h_f, h_m\}$. We obtain our calibrated parameter values by estimating these moments from the data and plugging them into the equations. We match an average male (female) labor supply of 42.5 (37.1) hours per week for France from our SHARE sample. As SHARE lacks detailed data on time use outside of the formal labor market, we estimate home production hours with data on hours spent on "unpaid domestic work" from the Multinational Time Use Study (MTUS), a harmonized collection of time diary data. We use data collected in 2009-10 for France and limit the sample to those aged 40-59 in which the individual and their partner works full-time to best approximate our sample used from SHARE. We match average male (female) home production of 11.9 (20.5) hours a week from the MTUS for France. Table 1 summarizes the baseline calibration for France.

This leaves only the endogenous bargaining power function $\theta(E)$. First note that specifying a functional form is not required for the baseline calibration as the *equilibrium* θ^* can be calibrated as if it were an exogenous parameter. However, when introducing informal care, the functional form of $\theta(E)$ plays a crucial role in the model's predictions with endogenous bargaining power. As is clear from Figure 3, any female labor supply equilibrium with informal care (lower than the baseline) can be rationalized in the model by selecting the appropriate change in bargaining power. Moreover, unlike preference parameters, there is no standard approach or directly applicable research to help credibly pin down the bargaining power function. As such, we choose a simple linear functional form:

$$\theta(E) = (0.5 - b)E + b. \tag{17}$$

This form implies $\theta(1) = 0.5$, or that equal earnings yields equal bargaining power. Denoting the baseline equilibrium earnings ratio E^*, it must that $\theta^*(E^*) = (0.5 - b)E^* + b$. As we have already pinned down θ^* and E^* (as we have matched male and female labor supply), this implies $b = \frac{\theta^* - 0.5E^*}{1 - E^*} = 0.25$.

In our examples below, we compare the model equilibrium with and without informal caregiving. Given the potential sensitivity of numerical results to the form of $\theta(E)$, we compare predictions from (17) with those from which bargaining power is exogenously held fixed at the baseline level: $\theta(E) = \theta^*$. Exogenous bargaining power can be viewed as the limiting case of the endogenous model, where the bargaining power curve becomes horizontal.

3.1.2 Informal care

We next compare equilibrium allocations with differing levels of informal caregiving requirements. Specifically, we compare the baseline of no informal care to an equilibrium with 15 hours of care per week. In addition to examining differences in consumption and time allocation patterns across partners, we are also interested in the distribution of welfare costs. We use a consumption-equivalent variation (CEV) measure to quantify the difference in welfare effects of informal caregiving across partners. Our welfare measure is akin to asking by what percentage market consumption has to be decreased (holding leisure and home consumption constant) to make an individual indifferent to the household providing informal care. Formally, welfare λ_i is defined by the condition:

$$u((1 + \lambda_i)c_i, d_i, l_i) = u(c_i^z, d_i^z, l_i^z)$$

where z superscripts denote equilibrium outcomes associated with household informal care level z. Under the assumed log preferences, the welfare condition may be explicitly written:

$$\lambda_i = \exp(u(c_i^z, d_i^z, l_i^z) - u(c_i, d_i, l_i)) - 1.$$

For example, a $\lambda_i = -0.1$ implies an individual would be indifferent between giving up 10% of baseline market consumption or their household providing informal care level z.

A summary of equilibrium results are provided in Table 2. Let us first focus on results under endogenous bargaining power. Recall that by Proposition 3, it is optimal for the female to provide all 15 hours of informal care. As a result, female labor supply is 6.1 hours (16.4%) lower and home production 1.1 hours (5.3%) lower with informal caregiving. The

Table 2: Equilibrium with and without informal care (France)

		Endogenous θ		Exogenous θ	
	No care	Informal care	% change	Informal care	% change
e_f	37.1	31.0	-16.4	26.7	-28.1
e_m	42.5	43.4	2.1	46.4	9.2
h_f	20.5	19.4	-5.3	19.3	-5.6
h_m	11.9	11.3	-5.3	11.2	-5.6
l_f	59.2	51.4	-13.2	55.8	-5.8
l_m	62.4	62.2	-0.4	59.2	-5.2
c_f	1.00	0.88	-12.0	0.94	-5.6
c_m	1.40	1.39	-0.5	1.32	-5.6
d_f	0.23	0.20	-12.0	0.21	-5.6
d_m	0.32	0.31	-0.5	0.30	-5.6
θ	0.42	0.39	-7.1	0.42	0.0
λ_f	.	-0.32	.	-0.16	.
λ_m	.	-0.01	.	-0.15	.
λ_m/λ_f	.	0.04	.	0.95	.

Notes: hours $e_f, e_m, h_f, h_m, l_f, l_m$ converted to weekly hours. Market and home consumption levels normalized: $c_f = 1$. Informal care of 15 hours per week.

remaining 7.8 hours devoted to care comes at the expense of female leisure time. While the male partner in the caregiving household does not provide any care, they increase their labor supply slightly (0.9 hours per week) and reduce home production the same proportion as the female (in accordance with Proposition 4 and condition (7)).

Net declines in market and home production lead to lower consumption levels with informal care. However, the lower female (and higher male) labor supply results in a lower earnings ratio and bargaining weight in the care providing household—$\theta = 0.39$ compared to 0.42 in the no care baseline. As the bargaining weight directly determines the household sharing rule, market and home consumption is significantly lower (12.0%) for the caregiving female, but only slightly lower (0.5%) for their male partner.

Substantially less leisure time and lower consumption levels for the caregiving female results in a welfare measure $\lambda_f = -0.32$. The female in the no care household would give up to 32% of her market consumption to avoid the equilibrium outcomes of the caregiving female. In contrast, the male partner would only give up 1% of his market consumption to avoid the informal care equilibrium. In other words, the welfare burden of informal caregiving to the male is only 4% that of their female partner.

Turning to the limiting case of exogenous bargaining power, the welfare burden is significantly more evenly distributed across partners. With no threat of lost bargaining power, the female caregiver provides 10.4 fewer hours to the labor market compared to the no care baseline. The male partner in the caregiving household supplies 3.9 more hours to the labor market—more than three times the increase with endogenous bargaining power. As a result, the leisure time cost associated with caregiving is much more evenly borne across partners. Moreover, as θ is held fixed, the sharing rule does not change and the percentage decline

in market and home consumption levels are exactly equal for both partners (5.6%). The associated welfare cost for the female is 16% of baseline market consumption compared to 15% for the male—a ratio of 0.95.

3.2 Cross-country

3.2.1 Calibration

Parameters governing preferences and home production are assumed to be common across all countries and are left at the values calibrated for France. However, we allow two parameters to vary across countries in our numerical exercise. First, we change relative productivity γ_c to match the aggregate wage ratio between females and males reported in our SHARE sample for each country. Second, we calibrate baseline bargaining power θ_c^* to match the ratio of female to male hours worked for each country (Proposition 2 ensures identification of this moment). Similar to France, for θ_c^* to be an equilibrium with endogenous bargaining power in country c, it must be that $\theta_c^*(E_c^*) = (0.5 - b_c)\, E_c^* + b_c$, where E_c^* is the baseline equilibrium earnings ratio in the country. This implies the power earnings function will differ across countries through parameter b_c. Differences in b_c across countries reflects differences in institutions, culture, laws, and other factors that map relative earnings into bargaining power.

3.2.2 Baseline features

The first four columns in Table 3 provide a comparison of average hours worked in the baseline model (no informal care) and the data for each country. Average female labor supply in our SHARE sample ranged from 30.0 hours a week in the Netherlands to 42.9 hours in Poland. There was less variation in male labor supply, ranging from 40.8 hours in the Netherlands to 48.3 in Israel. The gender gap in labor supply ranged from 2.5 hours a week in Estonia to 13.3 hours in Switzerland. Recall our baseline was calibrated to match relative labor supply between genders within each country. Overall, the baseline model also gives a reasonable approximation of average labor supply levels for males and females across countries (correlation coefficient of 0.82 for female labor supply and 0.12 for male).

The last five columns in Table 3 provide some additional features of the baseline calibration. Gender wage ratios γ (estimated directly from SHARE data) ranged from 0.71 in Germany to 0.91 in Sweden and Belgium. As expected, higher wage ratios are associated with higher female bargaining weights θ (correlation coefficient of 0.9). Females in a country with a large estimated gender wage gap generally have lower bargaining power than in countries with small wage gaps (e.g. $\theta = 0.40$ in Poland compared to $\theta = 0.49$ in Sweden). However, even countries with similar wage ratios can differ in calibrated bargaining weights based on observed labor supply gaps. Take the case of Switzerland and Italy. Both countries have the same gender wage ratio: $\gamma = 0.81$. However, the targeted labor supply ratio is 0.89 in Italy compared to 0.71 in Switzerland. In order to rationalize this difference within the structure of the current model, it must be that the baseline equilibrium bargaining weight is lower in Italy than in Switzerland. This lower θ results in more hour supplied by females in Italy and hence, rationalizes the observed higher labor supply ratio.

Table 3: Cross-country baseline (no informal care)

	Model fit				Baseline features				
	e_f		e_m						
	Data	Model	Data	Model	γ	θ	e_f/e_m	h_f/h_m	l_f/l_m
Austria	35.8	35.1	44.7	43.9	0.75	0.43	0.80	1.74	1.00
Germany	36.2	35.9	43.4	43.2	0.71	0.40	0.83	1.95	0.95
Sweden	37.7	37.3	43.2	42.7	0.91	0.49	0.87	1.20	1.04
Netherlands	30.0	33.3	40.8	45.3	0.75	0.44	0.74	1.77	1.05
Spain	38.1	36.9	44.1	42.7	0.81	0.44	0.86	1.52	0.99
Italy	38.0	37.5	42.9	42.3	0.81	0.44	0.89	1.52	0.97
France	37.1	37.1	42.5	42.5	0.76	0.42	0.87	1.72	0.95
Denmark	36.8	37.7	41.2	42.3	0.89	0.47	0.89	1.27	1.01
Greece	40.9	37.1	46.9	42.5	0.76	0.42	0.87	1.70	0.95
Switzerland	33.5	33.0	46.8	46.1	0.81	0.48	0.71	1.50	1.12
Belgium	35.5	36.0	43.3	44.0	0.91	0.50	0.82	1.20	1.09
Israel	38.3	35.1	48.3	44.2	0.81	0.46	0.79	1.52	1.04
Czech Republic	42.0	38.3	45.6	41.6	0.78	0.42	0.92	1.63	0.93
Poland	42.9	38.0	47.2	41.8	0.74	0.40	0.91	1.80	0.91
Estonia	39.9	38.8	42.5	41.3	0.77	0.41	0.94	1.66	0.91

Notes: hours e_f, e_m converted to weekly hours. Data estimates for SHARE sample aged 40-59, both partners working at least 20 hours a week, and neither partner providing informal care.

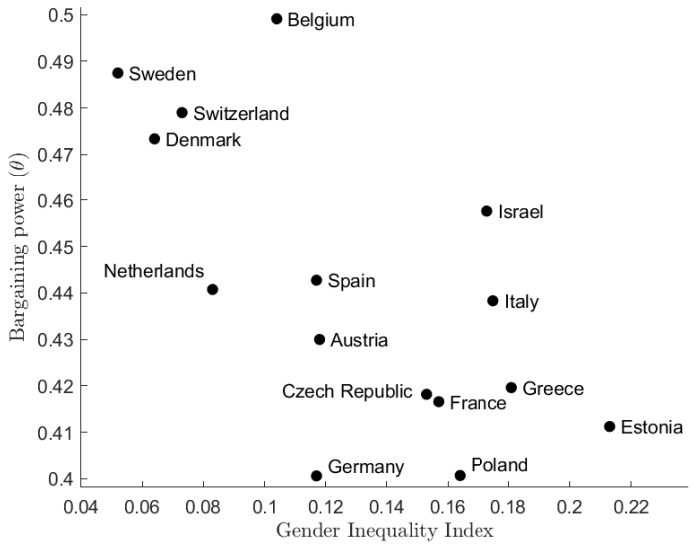


Figure 4: Bargaining power and Gender Inequality Index

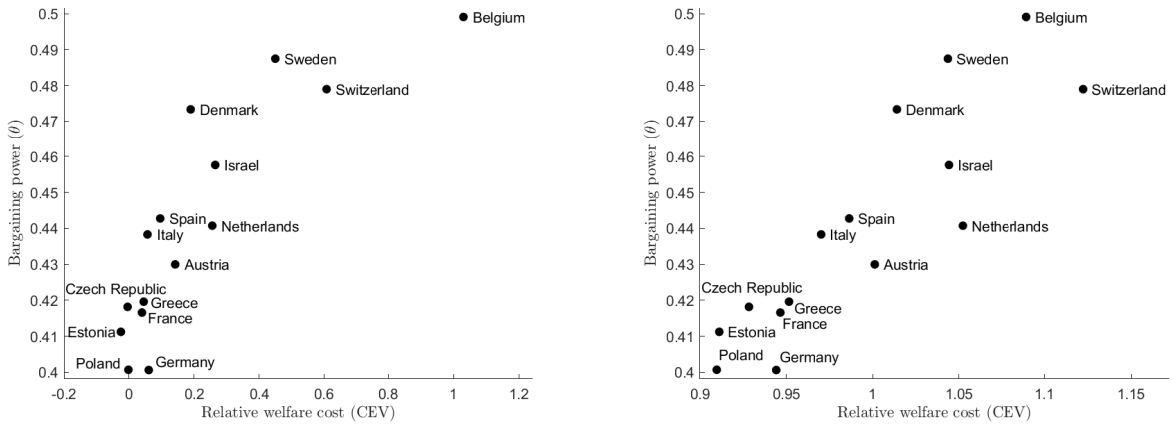
As an external comparison, Figure 4 plots our estimated bargaining weights against the United Nations Development Programme's Gender Inequality Index (GII). The GII is a composite measure that incorporates gender inequality on dimensions related to reproductive health, political and educational empowerment, and labor market participation. Notably, the GII suggests the calibrated bargaining weight may be somewhat too high in Belgium and too low in Germany and Poland. However, overall bargaining power maps reasonably well to the GII (correlation coefficient of -0.66).

Across all countries, the female partner supplies more hours to home production than the male—ranging from 20% more in Sweden to nearly double in Germany. In Switzerland, where female bargaining power is high but the wage ratio is about average, the female partner enjoys 12% higher leisure than the male. In contrast, low female bargaining power in Poland and Estonia results in a female leisure time equal to only 91% of her male partner.

3.2.3 Informal care

Figure 5 plots the relative welfare costs of 15 hours of informal care against each countries baseline bargaining weight. Results are provided for both endogenous and exogenous bargaining power. In either case, the welfare burden of informal care shifts towards the male partner as θ increases. Recall the relative welfare cost to the male in France was estimated at 4% of the female with endogenous bargaining power ($\lambda_m/\lambda_f = 0.04$). This estimate for France is included in panel (a) of the figure. In a few countries (e.g. Estonia), male welfare is slightly *higher* in the informal care equilibrium due to increased bargaining weight. In contrast, the relative burden is about half as high for males in Sweden and Switzerland and fully shared between genders in Belgium.

As demonstrated in panel (b), at high enough (fixed) bargaining power, the welfare burden of informal care is shifted disproportionately onto the male partner. This occurs even though the female continues to provide the entire amount of informal care. The decline



(a) Endogenous θ (b) Exogenous θ

Figure 5: Relative welfare cost of informal care (λ_m/λ_f)

in male leisure time driven by an increase in male labor supply is larger than the decline in female leisure associated with informal caregiving. It is because the male partner earns a higher return in the labor market that is optimal for him to shoulder a larger share of the welfare burden when bargaining power is fixed and relatively equal. However, as with France, when female bargaining power is relatively low, she will bare a larger share of the burden than the male partner.

3.3 Fixed labor supply

Table 4 revisits the equilibrium for France assuming 15 hours of informal care, holding male and female labor supply fixed at the baseline (no care) level. Results are also provided for Sweden as a complementary case. Note that with fixed labor supply, bargaining power remains constant regardless of whether it is exogenous or endogenously determined. In France, the non-negativity constraint on caregiving binds, and the female provides all 15 hours of care. In order to allow for this care without changing labor supply, she reduces home production by 4.3 hours and leisure by 11.7 hours. In this case, the French female in the no care equilibrium is willing to give 29% of her market consumption to avoid the caregiving equilibrium. This welfare cost falls in between those estimated with endogenous and exogenous bargaining power and flexible labor supply shown in Table 2.

In lieu of increasing labor supply, the French male increases home production in the presence of informal care. This results in a small 1.3% decline in leisure. He also suffers from the lower home consumption that accompanies lower female home production hours. On net, the welfare cost of caregiving to the French male is 6% of baseline market consumption, or about 20% the welfare loss of the female.

In Sweden, female bargaining power is high enough that caregiving is divided between partners—11.8 hours provided by the female and 3.2 hours by the male. As the informal care equilibrium is an interior solution for Sweden, condition (12) holds and home production is

Table 4: Fixed labor supply equilibrium with and without informal care

	France			Sweden		
	No care	Informal care	% change	No care	Informal care	% change
h_f	20.5	16.2	-21.1	17.1	14.1	-17.6
h_m	11.9	12.7	6.8	14.2	14.1	-0.7
l_f	59.2	48.5	-18.0	62.4	53.6	-14.0
l_m	62.4	61.6	-1.3	59.8	56.7	-5.2
θ	0.42	0.42	0.0	0.49	0.49	0.0
z_f	0.0	15.0	.	0.0	11.8	.
z_m	0.0	0.0	.	0.0	3.2	.
λ_f	.	-0.29	.	.	-0.24	.
λ_m	.	-0.06	.	.	-0.11	.
λ_m/λ_f	.	0.20	.	.	0.48	.

Notes: hours h_f, h_m, l_f, l_m converted to weekly hours. Informal care of 15 hours per week.

equal across partners.[4] Compared to France, the leisure cost of informal care is more evenly divided between partners as well. These allocations lead to a shift in the welfare cost of care from the female towards the male, with a welfare ratio of 48% for Sweden.

The comparison between France and Sweden highlights the importance of bargaining power in driving the relative welfare burden across partners when labor supply is held fixed. Figure 6 plots the welfare ratios for all countries in our sample. In the three countries with the highest welfare ratio—Belgium, Sweden, and Denmark—caregiving hours are divided between partners. In the remaining countries, care continues to fall entirely on the female.

4 Conclusions

With the aid of a simple collective model of intra-household bargaining we analyzed the time and consumption allocation decisions and welfare costs associated with informal caregiving. Our results suggest that with relatively equal bargaining power and fully flexible labor markets, the welfare burden of caregiving can be fairly evenly distributed across household partners. However, with endogenous bargaining power, the welfare burden can be greatly skewed towards the primary caregiver. This is due to a "triple burden" of market work, home production, and caregiving. Likewise, labor market rigidities can exacerbate the total welfare cost of informal caregiving to the household as well as the unequal distribution of the burden. This implies policies that promote flexibility, such as caregiver leave, could provide substantial relief, particularly to high intensity caregivers. While most existing empirical research has focused on labor market outcomes, our results also suggest that future work should examine the empirical link between caregiving and total time allocation patterns, including other forms of home production.

On the bargaining side, we adopted the collective approach of Chiappori (1992) allowing

[4] In our numerical exercise $\alpha = 0.5$, so condition (12) simplifies to $h_m = h_f$.

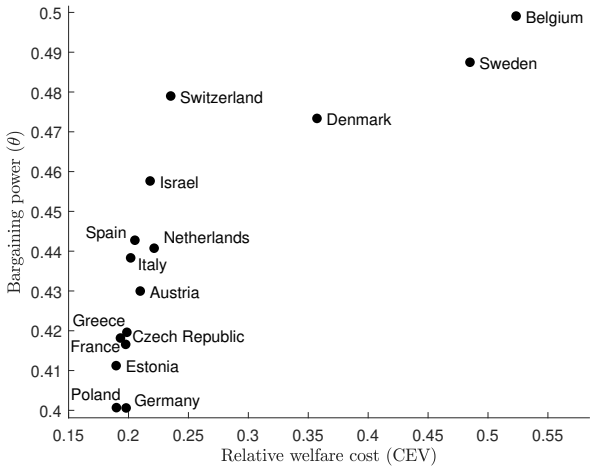


Figure 6: Fixed labor supply: relative welfare cost of informal care (λ_m/λ_f)

for a simple reduced form sharing rule. As an alternative, we could develop a more well defined cooperative Nash bargaining model of family partnerships (e.g. McElroy and Horney, 1981) where threat points are potentially endogenous to household decisions. This adds additional modeling complications but could allow for better identification of bargaining power response to changes in relative earnings. Moreover, it may be that threat points, and hence bargaining power, is influenced by potential earnings and not actual earnings (Pollak, 2005). This would suggest exogenous bargaining power as the relevant case study from our model. However, it could also be that lower labor supply decreases potential earnings through skill depreciation (or slower accumulation). For example, caregivers have reported an inability to reenter the job market after an extended period of time due to outdated knowledge (Carmichael et al., 2008). Moreover, results with labor market rigidities are effectively estimated with exogenous bargaining power. One could also consider such a model where informal caregiving requirements directly alters threat points. For example, if a daughter must assume responsibility of taking care of her elderly parent, it could lower the value of her outside option (e.g. divorce) and hence lower her equilibrium bargaining power. This could plausibly shift the welfare burden of caregiving towards the female similar to endogenous bargaining power.

While our simple static model demonstrates the potential quantitative influence of bargaining power, other considerations are warranted if robust counterfactual policy experiments are desired. Empirical evidence suggests informal caregiving lowers labor supply on both the extensive and intensive margins. Incorporating heterogeneous households and partially indivisible labor supply could yield additional insights. Dynamic considerations could also play an important role. For example, the expected persistence in informal caregiving shocks could have implications in the presence of labor market frictions or re-employment costs. It is also important to highlight that total welfare costs may be underestimated in our model if caregiving is accompanied by additional market expenses (e.g. food, medicine, etc.). Negative effects on caregiver health have also been well documented (Bauer and Sousa-Poza, 2015), suggesting the welfare burden may be further skewed towards those actually providing care.

Finally, while informal care continues to play a vital role in most countries, increased reliance on formal care markets is an important additional margin for consideration.

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Appendix: Proofs of propositions

Proposition 1

Denote the Lagrange multiplier on the household budget constraint λ_c and on the home production constraint λ_d. Then the first-order conditions for female and male market consumption are given by $\frac{(1-\theta)}{c_m} = \lambda_c$ and $\frac{\theta}{c_f} = \lambda_c$. These conditions combined with (2) yields (4). The first order conditions for domestic consumption are given by $\frac{(1-\theta)\kappa}{d_m} = \lambda_d$ and $\frac{\theta\kappa}{d_f} = \lambda_d$. These conditions combined with (3) yields (5).

Proposition 2

First-order conditions for e_f, e_m, h_f, h_m are given by:

$$\phi'\left(1 - e_f - h_f\right) = \frac{\gamma}{\theta\left(e_m + \gamma e_f\right)} \tag{18}$$

$$\phi'\left(1 - e_m - h_m\right) = \frac{1}{(1-\theta)\left(e_m + \gamma e_f\right)} \tag{19}$$

$$\phi'\left(1 - e_f - h_f\right) = \frac{\kappa\left(1-\alpha\right)h_f^{\eta-1}}{\theta\left(\alpha h_m^{\eta} + \left(1-\alpha\right)h_f^{\eta}\right)} \tag{20}$$

$$\phi'\left(1 - e_m - h_m\right) = \frac{\kappa\alpha h_m^{\eta-1}}{(1-\theta)\left(\alpha h_m^{\eta} + \left(1-\alpha\right)h_f^{\eta}\right)} \tag{21}$$

where we have used the implied sharing rule conditions: $c_f = \theta\left(e_m + \gamma e_f\right)$ and $d_f = \theta\left(\alpha h_m^{\eta} + \left(1-\alpha\right)h_f^{\eta}\right)^{\frac{1}{\eta}}$. Solving for $e_m\left(e_f, h_f\right)$ and $h_m\left(e_f, h_f\right)$, the system reduces to:

$$\phi'\left(1 - \varphi_1 h_f + \gamma e_f\right) = \frac{\kappa\left(1-\alpha\right)}{\gamma\left(1-\theta\right)\varphi_2 h_f} \tag{22}$$

$$\phi'\left(1 - e_f - h_f\right) = \frac{\kappa\left(1-\alpha\right)}{\theta\varphi_2 h_f} \tag{23}$$

where

$$\varphi_1 = \frac{\gamma\alpha}{\kappa(1-\alpha)}\left[\frac{\gamma\alpha}{1-\alpha}\right]^{\frac{\eta}{1-\eta}} + \frac{\gamma}{\kappa} + \left[\frac{\gamma\alpha}{1-\alpha}\right]^{\frac{1}{1-\eta}} > 0 \tag{24}$$

$$\varphi_2 = \alpha\left[\frac{\gamma\alpha}{1-\alpha}\right]^{\frac{\eta}{1-\eta}} + (1-\alpha) > 0. \tag{25}$$

Using these together with Cramer's rule yields

$$\frac{\partial e_f}{\partial\theta} = \frac{\left[\phi''(l_f)\frac{1}{\gamma(1-\theta)^2} + \phi''(l_m)\frac{\varphi_1}{\theta^2} - \frac{\kappa(1-\alpha)}{\gamma(1-\theta)^2\theta^2\varphi_2 h_f^2}\right]\frac{\kappa(1-\alpha)}{\varphi_2 h_f}}{(\gamma+\varphi_1)\phi''(l_m)\phi''(l_f) - \left(\phi''(l_f)\frac{1}{\gamma(1-\theta)} + \phi''(l_m)\frac{\gamma}{\theta}\right)\frac{\kappa(1-\alpha)}{\varphi_2 h_f^2}} < 0.$$

Moving to male labor supply, using conditions (18)-(21) and solving for $e_f(e_m, h_f)$ and $h_m(e_m, h_f)$, the system reduces to:

$$\phi'\left(1 - \left(1 + \frac{\varphi_2}{\gamma\kappa\alpha}\right)h_f + \frac{1}{\gamma}e_m\right) = \frac{\gamma\kappa\alpha}{\theta\varphi_2 h_f} \tag{26}$$

$$\phi'\left(1 - e_m - \left[\frac{\gamma\alpha}{1-\alpha}\right]^{\frac{1}{1-\eta}} h_f\right) = \frac{\kappa\alpha}{(1-\theta)\varphi_2 h_f} \tag{27}$$

where φ_2 is as defined in (25). Using these together with Cramer's rule yields

$$\frac{\partial e_m}{\partial\theta} = \frac{\left[-\phi''(l_f)\frac{1+\frac{\varphi_2}{\gamma\kappa\alpha}}{(1-\theta)^2} - \phi''(l_m)\left[\frac{\gamma\alpha}{1-\alpha}\right]^{\frac{1}{1-\eta}}\frac{\gamma}{\theta^2} + \frac{\gamma\kappa\alpha}{\theta^2(1-\theta)^2\varphi_2 h_f^2}\right]\frac{\kappa\alpha}{\varphi_2 h_f}}{\left(1 + \frac{\varphi_2}{\gamma\kappa\alpha} + \left[\frac{\gamma\alpha}{1-\alpha}\right]^{\frac{1}{1-\eta}}\frac{1}{\gamma}\right)\phi''(l_m)\phi''(l_f) - \left(\phi''(l_f)\frac{1}{\gamma(1-\theta)} + \phi''(l_m)\frac{\gamma}{\theta}\right)\frac{\kappa\alpha}{\varphi_2 h_f^2}} > 0.$$

Proposition 3

Assume $\gamma < 1$ and $e_f > 0$. Denote the Lagrange multiplier on the non-negativity constraint on female informal care λ_f, on male informal care λ_m, and on the constraint $z = z_f + z_m$ as λ_z. Then the first-order conditions for female and male informal care are given by $\theta\phi'(1 - e_f - h_f - z_f) = \lambda_f + \lambda_z$ and $(1-\theta)\phi'(1 - e_m - h_m - z_m) = \lambda_m + \lambda_z$. Combined this yields

$$\theta\phi'(1 - e_f - h_f - z_f) - \lambda_f = (1-\theta)\phi'(1 - e_m - h_m - z_m) - \lambda_m.$$

Given an interior solution for e_f, this condition may be rewritten using the first-order conditions for male and female labor supply as

$$(1-\gamma)\lambda_c + \lambda_{e_m} = \lambda_m - \lambda_f > 0 \tag{28}$$

where $\lambda_{e_m} \geq 0$ is the multiplier on the non-negativity constraint on e_m and we know that the household budget constraint holds with equality so $\lambda_c > 0$. Complementary slackness

implies $\lambda_m, \lambda_f \geq 0$ and feasibility require that at most one non-negativity constraint binds: $\lambda_m = 0$ or $\lambda_f = 0$. Thus it must be the case that $\lambda_m > 0$ and $\lambda_f = 0$ in order for (28) to hold. This shows that the female supplies all informal care. If $\gamma > 1$ and $e_m > 0$, equation (28) instead becomes

$$(\gamma - 1)\,\lambda_c + \lambda_{e_f} = \lambda_f - \lambda_m > 0.$$

Analogous arguments show that the male supplies all informal care in this case.

Proposition 4

With informal caregiving, the two equation system (22)-(23) becomes

$$\phi'\left(1 - \varphi_1 h_f + \gamma e_f\right) = \frac{\kappa\left(1-\alpha\right)}{\gamma\left(1-\theta\right)\varphi_2 h_f} \tag{29}$$

$$\phi'\left(1 - e_f - h_f - z_f\right) = \frac{\kappa\left(1-\alpha\right)}{\theta\varphi_2 h_f} \tag{30}$$

where φ_1 and φ_2 are as defined in (24) and (25). Using these together with Cramer's rule yields

$$\frac{\partial e_f}{\partial z} = \frac{\frac{\kappa(1-\alpha)}{\gamma(1-\theta)\varphi_2 h_f^2}\phi''\left(l_f\right) - \varphi_1\phi''\left(l_m\right)\phi''\left(l_f\right)}{\left(\gamma+\varphi_1\right)\phi''\left(l_m\right)\phi''\left(l_f\right) - \left(\phi''\left(l_f\right)\frac{1}{\gamma(1-\theta)} + \phi''\left(l_m\right)\frac{\gamma}{\theta}\right)\frac{\kappa(1-\alpha)}{\varphi_2 h_f^2}} < 0.$$

Moving to male labor supply, (26)-(27) become

$$\phi'\left(1 - \left(1 + \frac{\varphi_2}{\gamma\kappa\alpha}\right)h_f + \frac{1}{\gamma}e_m - z_f\right) = \frac{\gamma\kappa\alpha}{\theta\varphi_2 h_f}$$

$$\phi'\left(1 - e_m - \left[\frac{\gamma\alpha}{1-\alpha}\right]^{\frac{1}{1-\eta}} h_f\right) = \frac{\kappa\alpha}{\left(1-\theta\right)\varphi_2 h_f}$$

where φ_2 is as defined in (25). Using these together with Cramer's rule yields

$$\frac{\partial e_m}{\partial z} = \frac{\phi''\left(l_m\right)\phi''\left(l_f\right)\left[\frac{\gamma\alpha}{1-\alpha}\right]^{\frac{1}{1-\eta}} - \frac{\kappa\alpha}{(1-\theta)\varphi_2 h_f^2}\phi''\left(l_f\right)}{\left(1 + \frac{\varphi_2}{\gamma\kappa\alpha} + \left[\frac{\gamma\alpha}{1-\alpha}\right]^{\frac{1}{1-\eta}}\frac{1}{\gamma}\right)\phi''\left(l_m\right)\phi''\left(l_f\right) - \left(\phi''\left(l_f\right)\frac{1}{\gamma(1-\theta)} + \phi''\left(l_m\right)\frac{\gamma}{\theta}\right)\frac{\kappa\alpha}{\varphi_2 h_f^2}} > 0.$$

Moving to female home production, (29)-(30) can again be used with Cramer's rule to find

$$\frac{\partial h_f}{\partial z} = \frac{-\gamma\phi''\left(l_m\right)\phi''\left(l_f\right)}{\left(\gamma+\varphi_1\right)\phi''\left(l_m\right)\phi''\left(l_f\right) - \left(\phi''\left(l_f\right)\frac{1}{\gamma(1-\theta)} + \phi''\left(l_m\right)\frac{\gamma}{\theta}\right)\frac{\kappa(1-\alpha)}{\varphi_2 h_f^2}} < 0.$$

Finally, the combined optimally condition for home production remains identical to the case without informal caregiving: $h_m = \left[\frac{\gamma\alpha}{1-\alpha}\right]^{\frac{1}{1-\eta}} h_f$. Thus it is clear that $\frac{\partial h_m}{\partial h_f} > 0$ and hence $\frac{\partial h_m}{\partial z} > 0$. This also shows that the proportional response to increased informal caregiving will be the same for male and female home production.